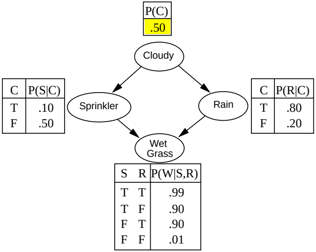
Problem #24

A typical belief network with conditional probabilities is given in the following figure:



The letters *C,R,S* and *W* stand for *Cloudy, Rain, Sprinkler*, and *Wet Grass*, respectively. All variables (nodes) are Boolean, so the probability of, say, $\neg A$ in any row of its table is $1 - P(A)$.

a) Assuming that the nodes are introduced in the following order *Wet Grass, Sprinkler, Rain* and *Cloudy* construct a corresponding belief network. Show which probabilities need to be specified.

b) Compute probabilities $\mathbf{P}(W)$ and $\mathbf{P}(S|W)$.

Problem #24: Solution a)

Order: W, S, R, C. Probabilities $\mathbf{P}(\mathbf{W})$, $\mathbf{P}(\mathbf{S}|\mathbf{W})$, $\mathbf{P}(\mathbf{R}|\mathbf{S},\mathbf{W})$, $\mathbf{P}(\mathbf{C}|\mathbf{R},\mathbf{S},\mathbf{W})$

- (W)

Problem #24: Solution a)

Order: W, S, R, C. Probabilities $\mathbf{P(W)}$, $\mathbf{P(S|W)}$, $\mathbf{P(R|S,W)}$, $\mathbf{P(C|R,S,W)}$

- (W)
- $P(S|W) = P(S)$?

Problem #24: Solution a)

Order: W, S, R, C. Probabilities $\mathbf{P}(\mathbf{W})$, $\mathbf{P}(\mathbf{S}|\mathbf{W})$, $\mathbf{P}(\mathbf{R}|\mathbf{S},\mathbf{W})$, $\mathbf{P}(\mathbf{C}|\mathbf{R},\mathbf{S},\mathbf{W})$

- (W)
- $P(S|W) = P(S)$?
- $P(S|W) = P(S)$? No!

Problem #24: Solution a)

Order: W, S, R, C. Probabilities $\mathbf{P(W)}$, $\mathbf{P(S|W)}$, $\mathbf{P(R|S,W)}$, $\mathbf{P(C|R,S,W)}$

- (W)
- $P(S|W) = P(S)$?
- $P(S|W) = P(S)$? No!
- $P(R|S,W) = P(R|S)$?

Problem #24: Solution a)

Order: W, S, R, C. Probabilities $\mathbf{P}(\mathbf{W})$, $\mathbf{P}(\mathbf{S}|\mathbf{W})$, $\mathbf{P}(\mathbf{R}|\mathbf{S},\mathbf{W})$, $\mathbf{P}(\mathbf{C}|\mathbf{R},\mathbf{S},\mathbf{W})$

- (W)
- $P(S|W) = P(S)$?
- $P(S|W) = P(S)$? No!
- $P(R|S,W) = P(R|S)$?
- $P(R|S,W) = P(R|S)$? No!

Problem #24: Solution a)

Order: W, S, R, C. Probabilities $\mathbf{P}(\mathbf{W})$, $\mathbf{P}(\mathbf{S}|\mathbf{W})$, $\mathbf{P}(\mathbf{R}|\mathbf{S},\mathbf{W})$, $\mathbf{P}(\mathbf{C}|\mathbf{R},\mathbf{S},\mathbf{W})$

- (W)
- $P(S|W) = P(S)$?
- $P(S|W) = P(S)$? No!
- $P(R|S,W) = P(R|S)$?
- $P(R|S,W) = P(R|S)$? No!
- $P(C|R,S,W) = P(C|R,S)$? $P(C|R,S,W) = P(C|R)$?
- $P(C|R,S,W) = P(C|R,S)$? Yes! $P(C|R,S,W) = P(C|R)$? No!

Problem #24: Solution b)

$$\mathbf{P(W)} = \sum_c \sum_s \sum_r \mathbf{P(c)P(s|c)P(r|c)P(W|s,r)}$$

$$\mathbf{P(W)} = \sum_c \mathbf{P(c)} \sum_s \mathbf{P(s|c)} \sum_r \mathbf{P(r|c)P(W|s,r)}$$

$$\begin{aligned}
P(W) &= P(c)\{P(s|c)[P(r|c)P(w|s,r) + P(\neg r|c)P(w|s\neg r)] + \\
&\quad P(\neg s|c)[P(r|c)P(w|\neg s,r) + P(\neg r|c)P(w|\neg s\neg r)]\} + \\
&\quad P(\neg c)\{P(s|\neg c)[P(r|\neg c)P(w|s,r) + P(\neg r|\neg c)P(w|s\neg r)] + \\
&\quad P(\neg s|\neg c)[P(r|\neg c)P(w|\neg s,r) + P(\neg r|\neg c)P(w|\neg s\neg r)]\} \\
P(W) &= 0.5\{0.1[0.8*0.99 + 0.2*0.9] + 0.9[0.8*0.9 + 0.2*0.01]\} + \\
&\quad 0.5\{0.5[0.2*0.99 + 0.8*0.9] + 0.5[0.2*0.9 + 0.8*0.01]\} \\
P(W) &= 0.5[0.1*1.512 + 0.9*0.722] + 0.5[0.5*0.918 + 0.5*0.188] \\
&\quad 0.5*0.801 + 0.5*0.553 = 0.677 \\
P(\neg W) &= 0.323
\end{aligned}$$

Problem #24: Solution b)

$$\mathbf{P(S|W) = P(S,W)/P(W)} = \alpha \sum_c \sum_r \mathbf{P(c)P(S|c)P(r|c)P(W|S,r)}$$

$$\mathbf{P(S|W)} = \alpha \sum_c \mathbf{P(c)P(S|c)} \sum_r \mathbf{P(r|c)P(W|S,r)}$$

$$\begin{aligned} P(S|W) &= \alpha\{P(c)P(s|c)[(P(r|c)P(w|s,r) + P(\neg r|c)P(w|s,\neg r)] + \\ &\quad P(\neg c)P(s|\neg c)[(P(r|\neg c)P(w|s,r) + P(\neg r|\neg c)P(w|s,\neg r)]\} \\ &= \alpha[0.05(0.8*0.99+0.18)+0.25(0.2*0.99+0.72)] = 0.2781 \\ P(\neg S|W) &= \alpha[0.45(0.8*0.9+0.2*0.01)+0.25(0.2*0.9+0.8*0.01)] \\ P(S|\neg W) &= ? \\ P(\neg S|\neg W) &= ? \end{aligned}$$

$$P(S|W) + P(\neg S|W) = 1, \quad P(S|\neg W) + P(\neg S|\neg W) = 1,$$