CS483-09 Elementary Graph Algorithms

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Office hours: Tue. & Thur. 1:30pm - 2:30pm or by appointments

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http://www.cs.gmu.edu/∼ lifei/teaching/cs483_fall07/

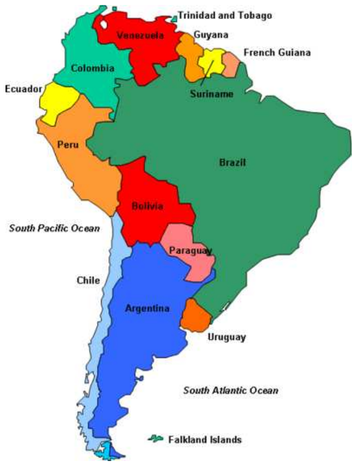
Based on “Introduction to Algorithms” by T. Cormen, C. Leiserson, R. Rivest, and C. Stein and

“Algorithms” by S. Dasgupta, C. Papadimitriou, and U. Vazirani.

Outline

- Representation of Graphs
- Breath-first Search
- Depth-first Search
- Topological Sort

Why Graphs?



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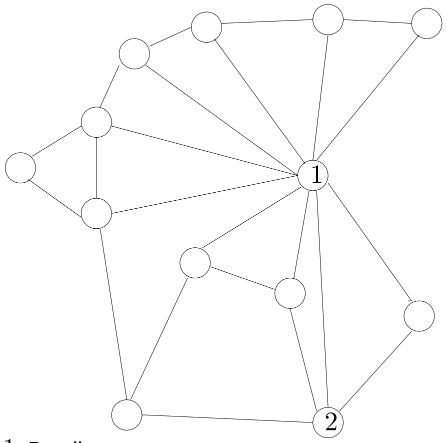
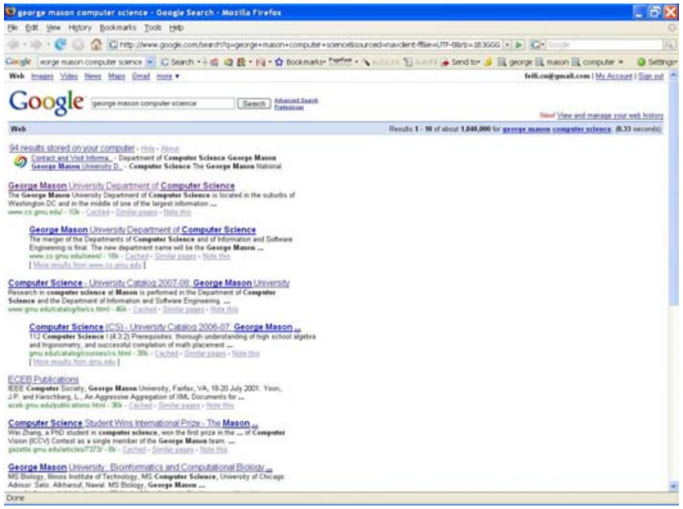
Why Graphs?



1 Brazil
2 Argentina

Why Graphs?



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MS Biology, Illinois Institute of Technology, MS Computer Science, University of Chicago Advisor: Saleh, Alkharouf, Nawal. MS Biology, George Mason ...

Done

Why Graphs?

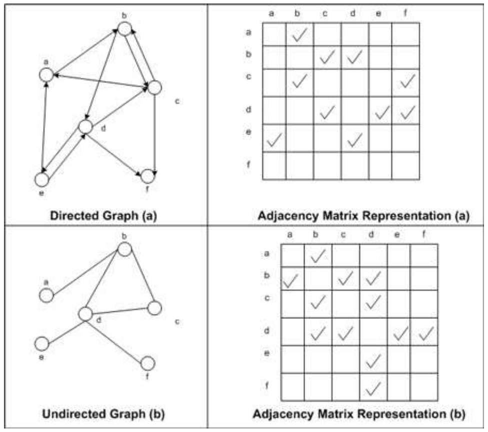
http://adcentered.typepad.com/photos

Graphs

- A graph $G = (V, E)$ is specified by a set of vertices (nodes) V and edges E between selected pairs of vertices.
- Edges are symmetric — *undirected graph*
- Directions over edges — *directed graph*
- Examples: political maps, exam conflicts, World Wide Web, etc.

Graph Representation

- Adjacency-matrix



Directed Graph (a)

	a	b	c	d	e	f
a		✓				
b			✓	✓		
c		✓				✓
d			✓		✓	✓
e	✓			✓		
f						

Adjacency Matrix Representation (a)

Undirected Graph (b)

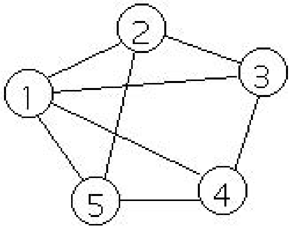
	a	b	c	d	e	f
a		✓				
b	✓		✓	✓		
c		✓		✓		
d		✓	✓		✓	✓
e				✓		
f				✓		

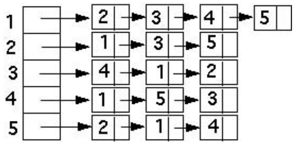
Adjacency Matrix Representation (b)

http://msdn2.microsoft.com/en-us/library/

Graph Representation

- Adjacency-list





1 → 2 → 3 → 4 → 5
2 → 1 → 3 → 5
3 → 4 → 1 → 2
4 → 1 → 5 → 3
5 → 2 → 1 → 4

Graph Traversal is Important

Exploring a graph is rather like navigating a maze.

Which parts of the graph are reachable from a given vetex?

`http://www.sheffordtown.co.uk/maze/index.html`

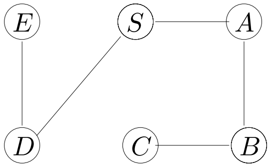
Outline

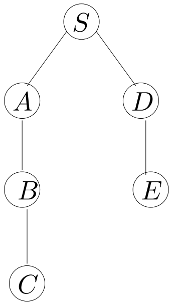
- Representation of Graphs
- Breath-first Search
- Depth-first Search
- Topological Sort

Breath-first Search (BFS)

BFS

1. Identifies all the vertices of a graph that can be reached from a designated starting point, and
2. Finds explict paths via a depth-first search tree.





Breath-first Search (BFS)

Input: Graph $G = (V, E)$, directed or undirected; vertex $s \in V$

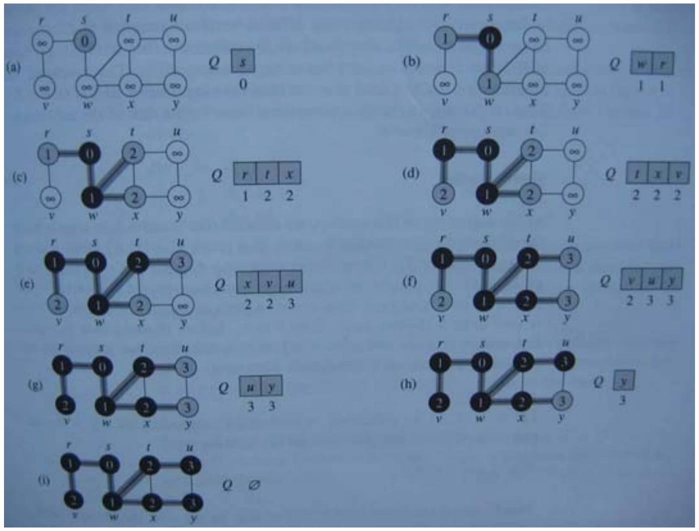
Output: For all vertices u reachable from s, $d(u)$ is set to the distance from s to u

Intuition: Proceed layer by layer

Algorithm 0.1: BFS(G, s)

```
for $\forall u \in V$
  $d(u) = \infty$
$d(s) = 0$

$Q = [s]$
while $Q \neq \emptyset$
  $u =$ Pop $(Q)$
  for $(u, v) \in E$
    if $d(v) = \infty$
      then Push $(Q, v)$
           $d(v) = d(u) + 1$
```



Breath-first Search (BFS)

- The correctness proof: Use an induction method
- The overall running time of BFS is $O(|V| + |E|)$.
  - Each vertex is put on the queue exactly once, when it is first encountered, so there are $2 \cdot |V|$ queue operations.
  - Over the course of execution, this loop looks at each edge once (in directed graphs) or twice (in undirected graphs), and therefore takes $O(|E|)$ time.

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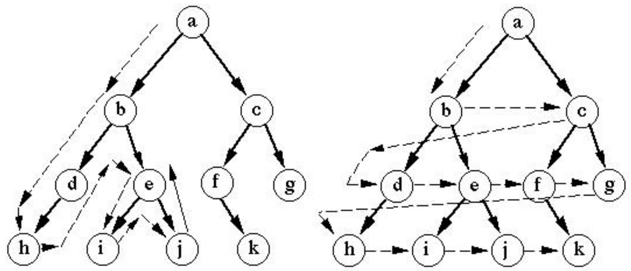
Depth-first Search

Input: Graph $G = (V, E)$, directed or undirected; vertex $s \in V$

Output: All vertices u reachable from s in timestamps of visiting

Intuition: Explore each vertex as much as you can



Depth-first search

Breadth-first search

http://www.cse.unsw.edu.au/ billw/Justsearch1.gif

Depth-first Search (DFS)

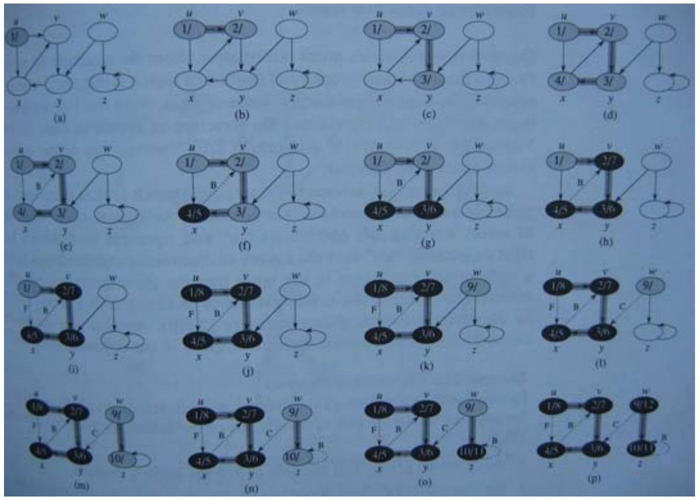
$\pi[u]$: the parent of a node u.

$time[u]$: timestamp when u is first discovered.

```
Algorithm 0.2: DFS($G(V, E)$)
 for each vertex $u \in V(G)$
   do color$[u] \leftarrow$ WHITE
        $\pi[u] \leftarrow$ NIL
 time $\leftarrow 0$
 for each vertex $u \in V(G)$
   do if color$[u] =$ WHITE
   then DFS-VISIT(u)
```

Algorithm 0.3: DFS-VISIT(u)

```
color$[u] \longleftarrow$ GRAY
$//$White vertex u has just been discovered.
$d[u] \longleftarrow$ time $\longleftarrow$ time $+ 1$
for each $v \in$ Adj$[u]$
$//$Explore edge $(u, v)$.
  do if color$[v] =$ WHITE
  then $\pi(u) \longleftarrow u$
  DFS-VISIT(v)
color$[u] \longleftarrow$ BLACK
$//$Blacken u; it is finished.
$d[u] \longleftarrow$ time $\longleftarrow$ time $+ 1$
```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