CS483 Design and Analysis of Algorithms*

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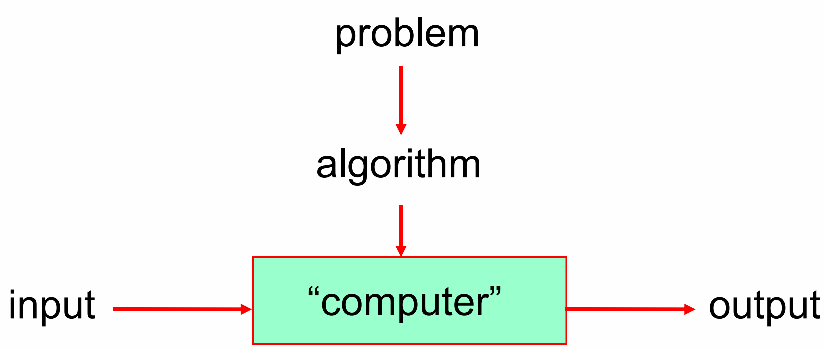
*This lecture note is based on *Introduction to The Design and Analysis of Algorithms* by Anany Levitin and Jyh-Ming Lie's cs483 notes.

Overview

- Introduction to algorithms
- Course syllabus

What is an algorithm?

- An **algorithm** is a sequence of unambiguous instructions for solving a problem, i.e., for obtaining a required output for any legitimate input in a finite amount of time.



Procedure of solving a problem on a computer

- Analyze and model a real problem as a computational problem
- Get the intuition
- **Design an algorithm**
  - Prove its correctness
- **Analyze the solution**, i.e., time efficiency, space efficiency, optimality, etc.
  - Can we get an improved solution?
  - Can we generalize our solution?
- Code an algorithm

Example of a computational problem

- Statement of problem:
  - Rank students based on their grades
- *Input:* A sequence of n numbers $<a_1, a_2, \ldots, a_n>$
- *Output:* A reordering of the input sequence $<a'_1, a'_2, \ldots, a'_n>$ so that $a'_i \leq a'_j$ whenever $i < j$
- *Algorithms:*
  - Selection sort
  - Insertion sort
  - Merge sort
  - (many others)

Selection Sort

- *Input:* An array a[1],…,a[n]
- *Output:* An array sorted in non-decreasing order
- *Algorithm:*

```
for i=1 to n
    swap a[i] with smallest of a[i],…a[n]
```

- *Example:* <5,3,2,8,3> $\rightarrow$ <2,3,3,5,8>

An algorithm

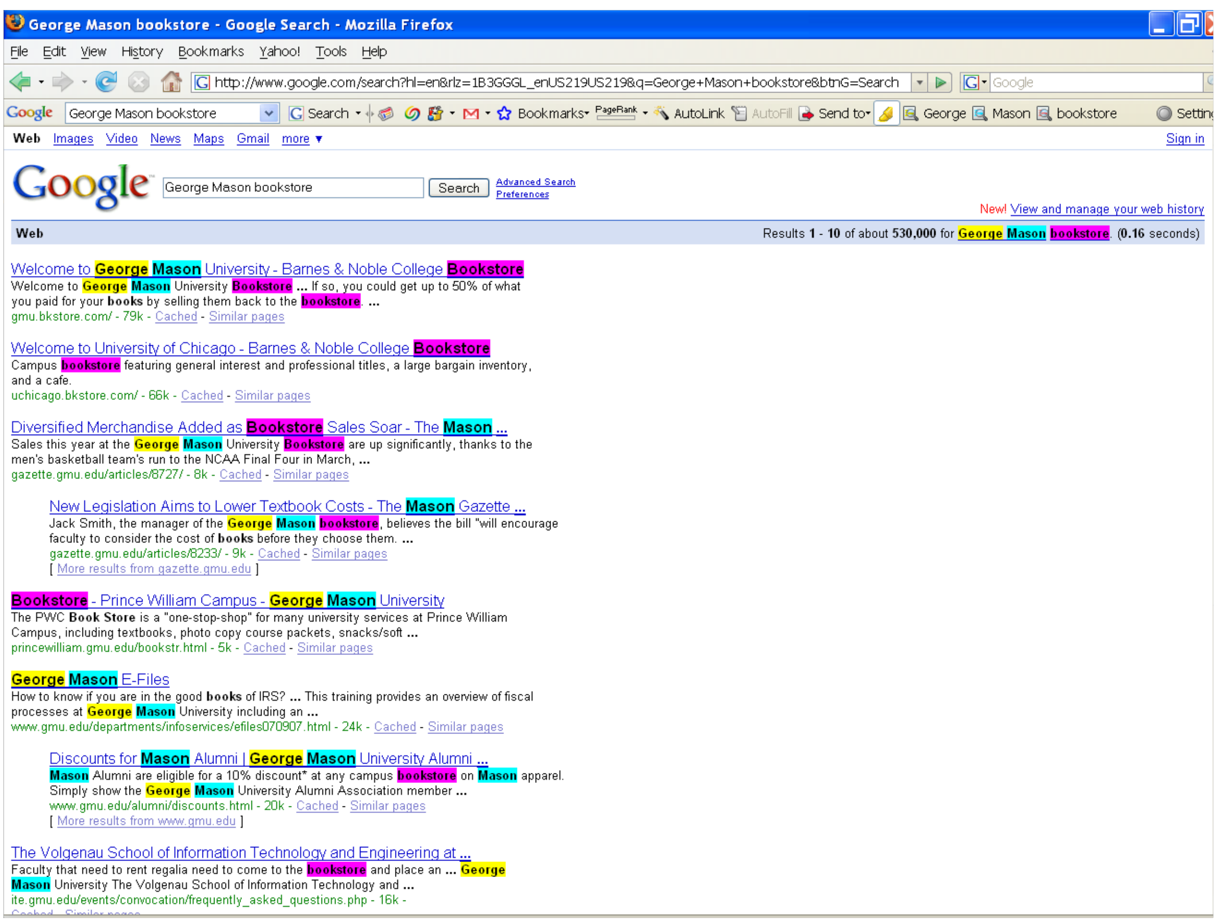
- Recipe, process, method, technique, procedure, routine,… with following requirements:
  - Finiteness
    - terminates after a finite number of steps
  - Definiteness
    - rigorously and unambiguously specified
  - Input
    - valid inputs are clearly specified
  - Output
    - can be proved to produce the correct output given a valid input
  - Effectiveness
    - steps are sufficiently simple and basic

Why study algorithms?

- Theoretical importance
  - The core of computer science
- Practical importance
  - A practitioner's toolkit of known algorithms
  - Framework for designing and analyzing algorithms for new problems

Example1 – String Matching (Chap. 3 and 7)

- A string is a sequence of characters from an alphabet.
- Problem: search strings in a text
- *Input:*
  - a string of m characters called the pattern
  - a string of n characters called the text
- *Output:*
  - a substring of the text that matches the pattern.



Example2 – Travelling Salesman Problem (TSP) (Chapter 3)

- Problem: Find the shortest tour through a given set of cities, which a salesman visits each city exactly once before returning to the starting city
- *Input:*
  - A map of n cities
  - Starting city
- *Output:*
  - The shortest tour which has all the cities

Travelling Salesman Problem

Weighted graph

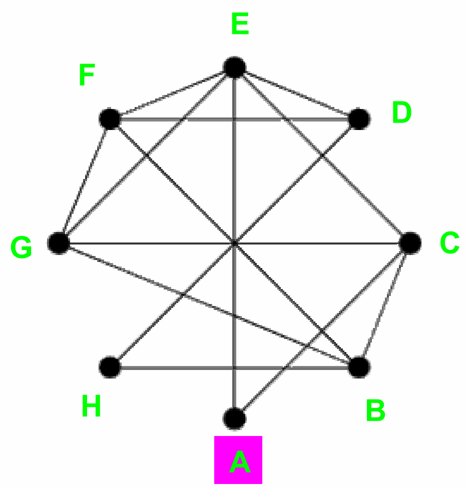


Image from Wolfram MathWorld

Travelling Salesman Problem

A→C → G → F → B → H → D → E → A

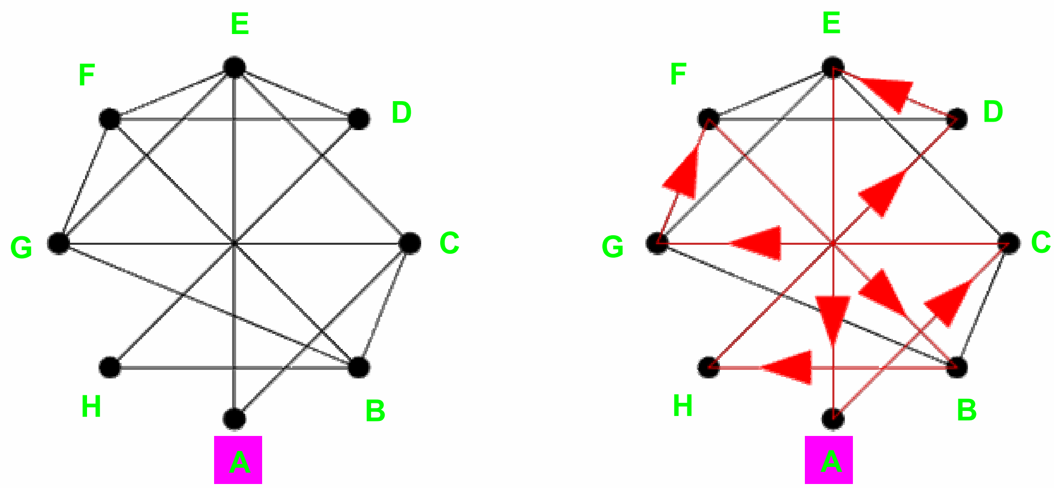
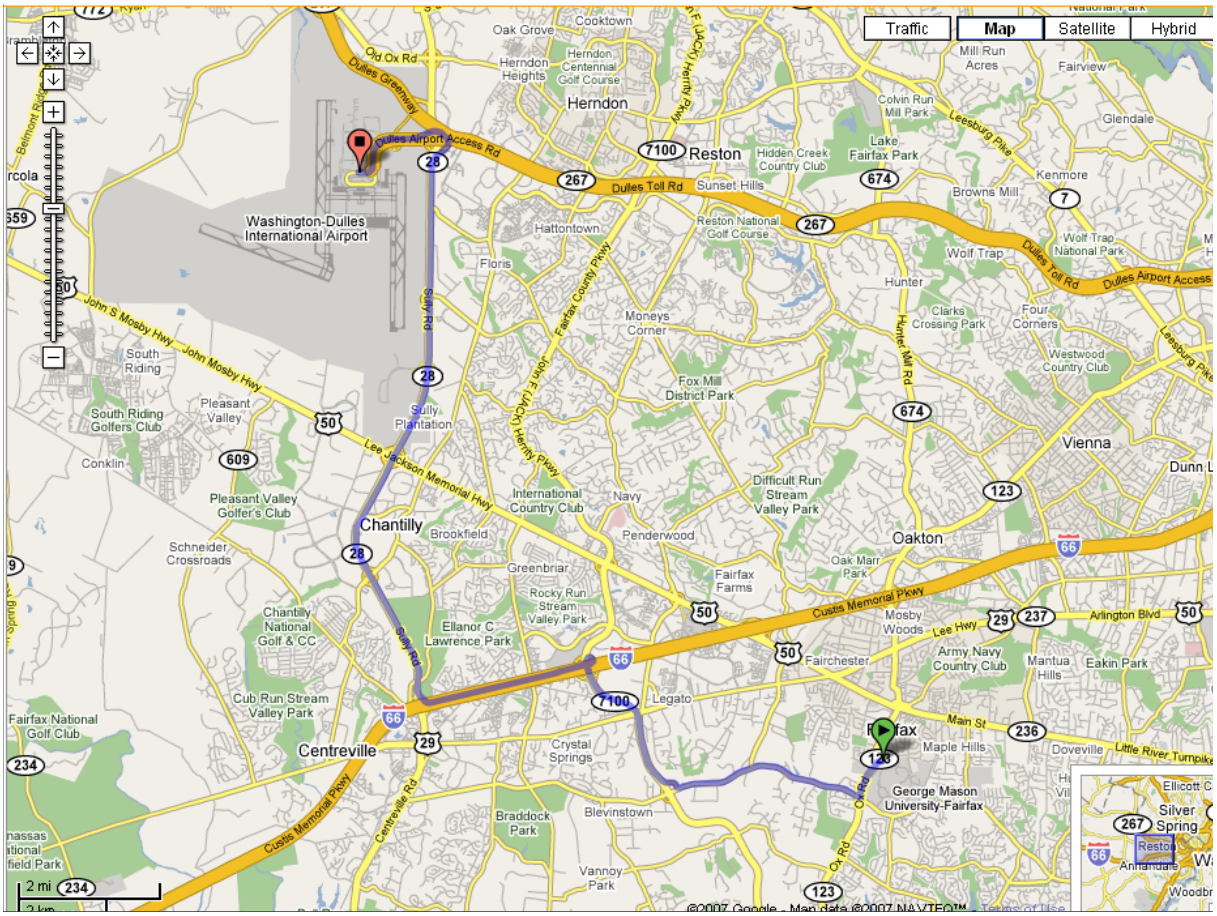


Image from Wolfram MathWorld

Example3 – Path Finding (Chap. 9)

- Problem: Find the optimal path from the origin to the destination subject to certain objectives
- *Input:*
  - A weighted graph
  - Origin and destination
- *Output:*
  - Optimal path



Example4 – Interval Scheduling (Chap. 8 and 9)

- Problem: Maximize the ***maximum number or possible size*** of requests.
- *Input:*
  - A shared resource used by one person at one time
  - A bunch of requests
    - User i: Can I reserve the resource (classroom, book, supercomputer, microscope, ..) from time s_i to f_i?
- *Output:*
  - A selection of requests with assigned resource

Example5 – Stable Marriage (Chap. 10)

- A set of marriages is *stable* if there are no two people of opposite sex who would both rather have each other than their current partners.
- Problem: Find a stable marriage matching for given men and women to be paired off in marriages.
- *Input:*
  - n men and n women
  - Each person has ranked all members of the opposite sex with a unique number between 1 and *n* in order of preference
- *Output:*
  - A matching

Basic issues related to algorithms

- How to design algorithms
- How to express algorithms
- Proving correctness
- Efficiency
  - Theoretical analysis
  - Empirical analysis
- Optimality and improvement

Greatest Common Divisor Problem

- ***Problem:*** Find gcd(*m,n*), the greatest common divisor of two nonnegative, not both zero integers *m* and *n*
- ***Examples:*** gcd(60,24) = 12, gcd(60,0) = 60

Solution 1

- Observation: gcd(m,n) ≤ min{m,n}
- *Consecutive integer checking algorithm*
  - Step 1 Assign the value of min{*m,n*} to *t*
  - Step 2 Divide *m* by *t.* If the remainder is 0, go to Step 3; otherwise, go to Step 4
  - Step 3 Divide *n* by *t.* If the remainder is 0, return *t* and stop; otherwise, go to Step 4
  - Step 4 Decrease *t* by 1 and go to Step 2

Solution 2

- *Middle-school procedure*
  - Step 1 Find the prime factorization of *m*
  - Step 2 Find the prime factorization of *n*
  - Step 3 Find all the common prime factors
  - Step 4 Compute the product of all the common prime factors and return it as gcd*(m,n)*
- Example: gcd(60,24)
  - m = 60 = **2** x **2** x **3** x 5
  - n = 24 = **2** x **2** x 2 x **3**
  - gcd(m, n) = gcd(60,24) = **2** x **2** x **3** = 12
- Not an algorithm! *Prime factorization*

Prime Factorization

- *Input:* Integer $n \geq 2$
- *Output:* A sequence of prime numbers S, whose multiplication is n.
- *Algorithm:*

```
find a list of prime numbers P that are smaller than n
i ← 2
while i < n do
      if n%i = 0
         then s ← i, n ← n/I
         else i ← next prime number
```

Sieve

- *Input:* Integer $n \geq 2$
- *Output:* List of primes less than or equal to n
- *Algorithm:*

for $p \leftarrow 2$ to n do $A[p] \leftarrow p$

for $p \leftarrow 2$ to $\lfloor n \rfloor$ do

    if $A[p] \neq 0$ //p hasn't been previously eliminated from the list

      $j \leftarrow p * p$

      while $j \leq n$ do

          $A[j] \leftarrow 0$ //mark element as eliminated

          $j \leftarrow j + p$

Sieve (cont.)

- Example

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	3		5		7		9		11		13		15		17		19	
2	**3**		5		7				11		13				17		19	
2	3		**5**		7				11		13				17		19	

Solution 3 - Euclid's Algorithm

- Euclid's algorithm is based on repeated application of equality

  $$\gcd(m,n) = \gcd(n,\ m \bmod n)$$

  until the second number becomes 0, gcd(m, 0) = 0 .
- Example: gcd(60,24) = gcd(24,12) = gcd(12,0) = 12
- *Algorithm*

```
while n ≠ 0 do
    r ← m mod n
    m ← n
    n ← r
return m
```


Algorithm design techniques/strategies

- Brute force
- Divide and conquer
- Decrease and conquer
- Transform and conquer
- Space and time tradeoffs
- Greedy approach
- Dynamic programming
- Iterative improvement
- Backtracking
- Branch and bound

Analysis of algorithms

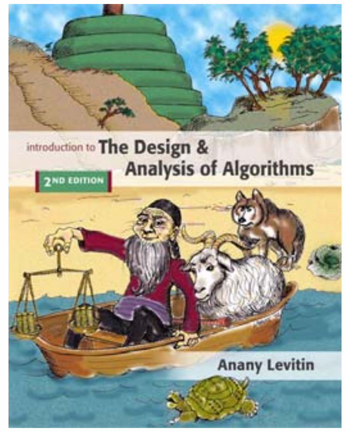
- How good is the algorithm?
  - time efficiency
  - space efficiency
- Does there exist a better algorithm?
  - Simplicity
  - Generality
  - lower bounds
  - optimality

Syllabus

- Lecture time
  - Tue & Thu 3:00-4:15pm
- Office Hour
  - Tue & Thu 4:30-5:30pm
  - Office: 443 ST II
- Course webpage:
  - www.cs.gmu.edu/~lifei/teaching/cs483_fall07/

Syllabus (cont.)

- TA: Yanyan Lu
  - Email: ylu4@gmu.edu
  - Office hour: Wed & Friday 4:00pm – 5:00pm
  - Room 437 STII
- Required Textbook:
  - ***Introduction to the Design and Analysis of Algorithms*** by Anany Levitin , Addison Wesley; 2nd edition (2007)



Syllabus (cont.)

- Topics
  - Analysis of Algorithm Efficiency
  - Brute Force
  - Divide (decrease) and Conquer
  - Transform and Conquer
  - Greedy Techniques
  - Dynamic Programming
  - Iterative Improvement
  - Limitations of Algorithm Power and Coping with Limitations

Syllabus (cont.)

- Grading (tentative)
  - Biweekly assignment (40%)
    - Work on your assignments independently.
    - List all the resources such as web, books and other students that may have helped with your solution.
    - Hand in hard copies.
    - One late submission (up to one week past the due date) per person per semester is permitted.
  - Midterm exam (25%)
  - Final exam (35%)
    - Two pages (letter size) of notes are allowed for both exams.

Some Suggestions

- Start working on assignments early
- Review notes and textbook after class
- Ask questions!

- Before next class
  - Read Chapter 1.1, 1.2, 1.4 and Appendix A.

- Next class
  - Algorithm analysis
  - Recursion