CS483 Design and Analysis of Algorithms*

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August 28, 2007

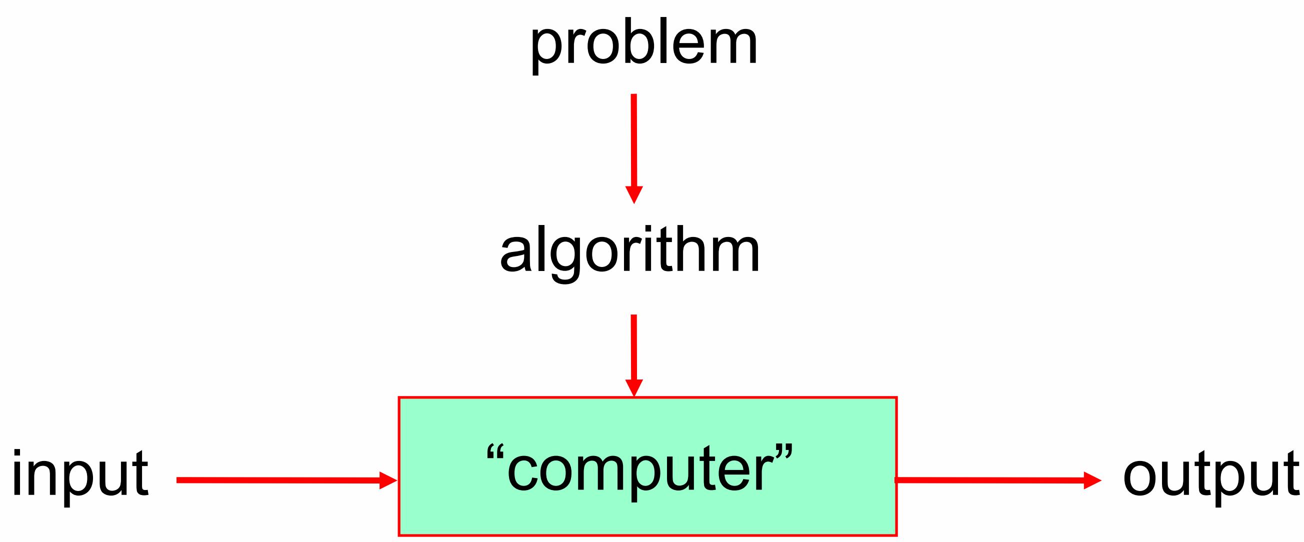
*This lecture note is based on *Introduction to The Design and Analysis of Algorithms* by Anany Levitin and Jyh-Ming Lie's cs483 notes.

Overview

- Introduction to algorithms
- Course syllabus

What is an algorithm?

- An **algorithm** is a sequence of unambiguous instructions for solving a problem, i.e., for obtaining a required output for any legitimate input in a finite amount of time.



Procedure of solving a problem on a computer

- Analyze and model a real problem as a computational problem
- Get the intuition
- **Design an algorithm**
  - Prove its correctness
- **Analyze the solution**, i.e., time efficiency, space efficiency, optimality, etc.
  - Can we get an improved solution?
  - Can we generalize our solution?
- Code an algorithm

Example of a computational problem

- Statement of problem:
  - Rank students based on their grades

- *Input:* A sequence of *n* numbers $<a_1, a_2, \ldots, a_n>$
- *Output:* A reordering of the input sequence $<a'_1, a'_2, \ldots, a'_n>$ so that $a'_i \leq a'_j$ whenever $i < j$
- *Algorithms:*
  - Selection sort
  - Insertion sort
  - Merge sort
  - (many others)

Selection Sort

- *Input:* An array a[1],…,a[n]
- *Output:* An array sorted in non-decreasing order
- *Algorithm:*

> **for *i*=1 to *n***
>   **swap a[*i*] with smallest of a[i],…a[*n*]**

- *Example:* <5,3,2,8,3> → <2,3,3,5,8>

An algorithm

- Recipe, process, method, technique, procedure, routine,… with following requirements:
  - Finiteness
    - terminates after a finite number of steps
  - Definiteness
    - rigorously and unambiguously specified
  - Input
    - valid inputs are clearly specified
  - Output
    - can be proved to produce the correct output given a valid input
  - Effectiveness
    - steps are sufficiently simple and basic

Why study algorithms?

- Theoretical importance
  - The core of computer science
- Practical importance
  - A practitioner’s toolkit of known algorithms
  - Framework for designing and analyzing algorithms for new problems

Example1 – String Matching (Chap. 3 and 7)

- A string is a sequence of characters from an alphabet.
- Problem: search strings in a text

- *Input:*
  - a string of m characters called the pattern
  - a string of n characters called the text
- *Output:*
  - a substring of the text that matches the pattern.

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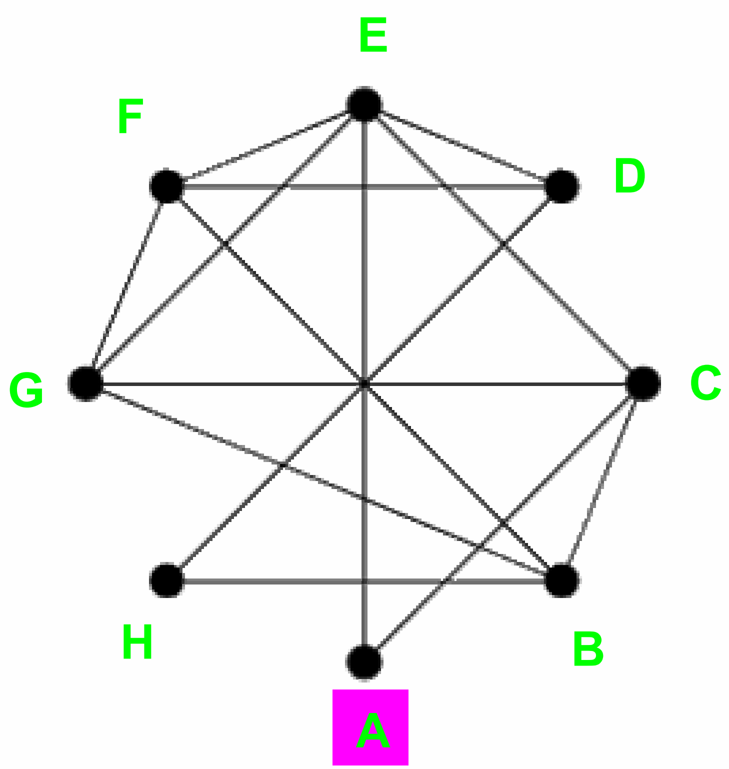
Example2 – Travelling Salesman Problem (TSP) (Chapter 3)

- Problem: Find the shortest tour through a given set of cities, which a salesman visits each city exactly once before returning to the starting city
- *Input:*
  - A map of n cities
  - Starting city
- *Output:*
  - The shortest tour which has all the cities

Travelling Salesman Problem

Weighted graph



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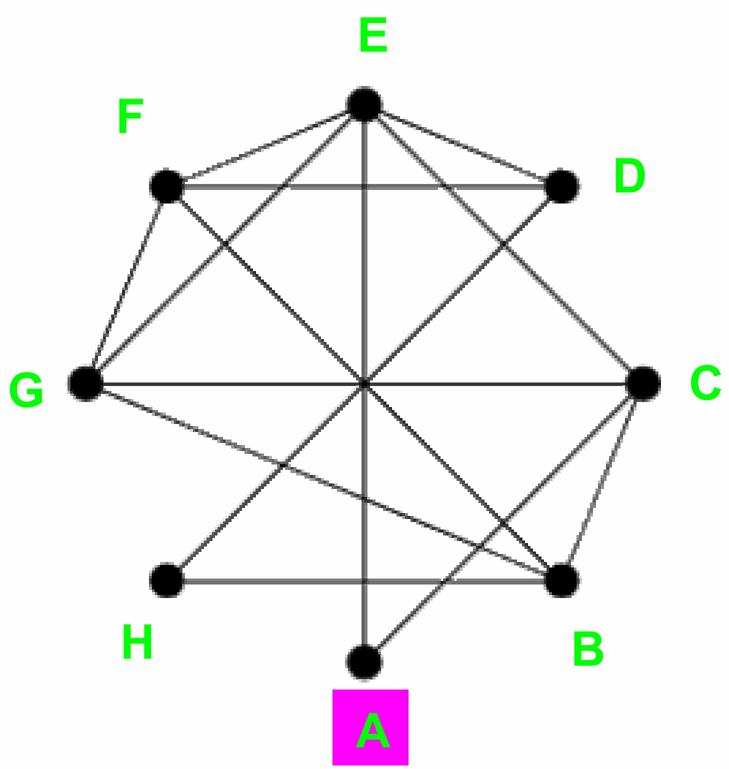
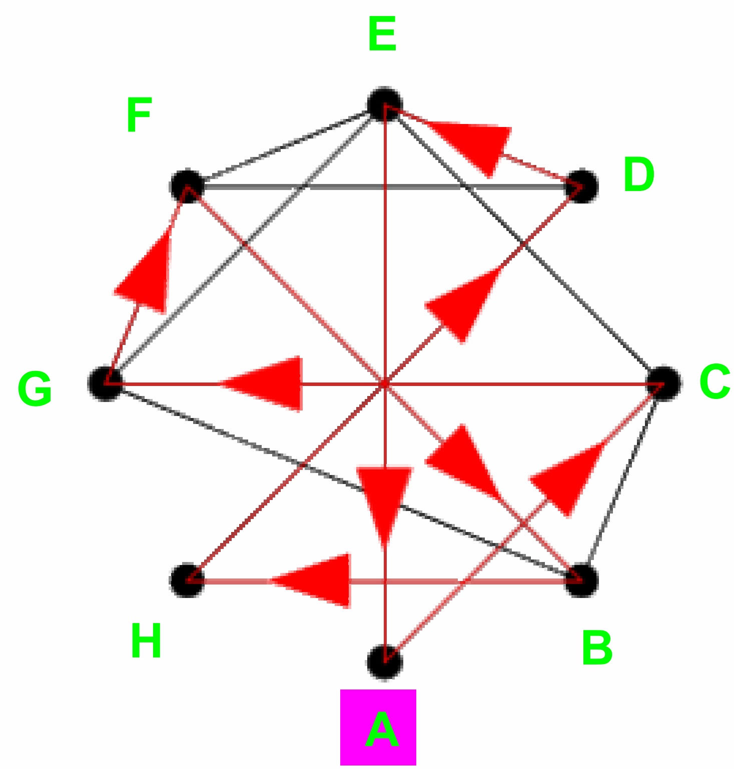
Travelling Salesman Problem

A→C → G → F → B → H → D → E → A





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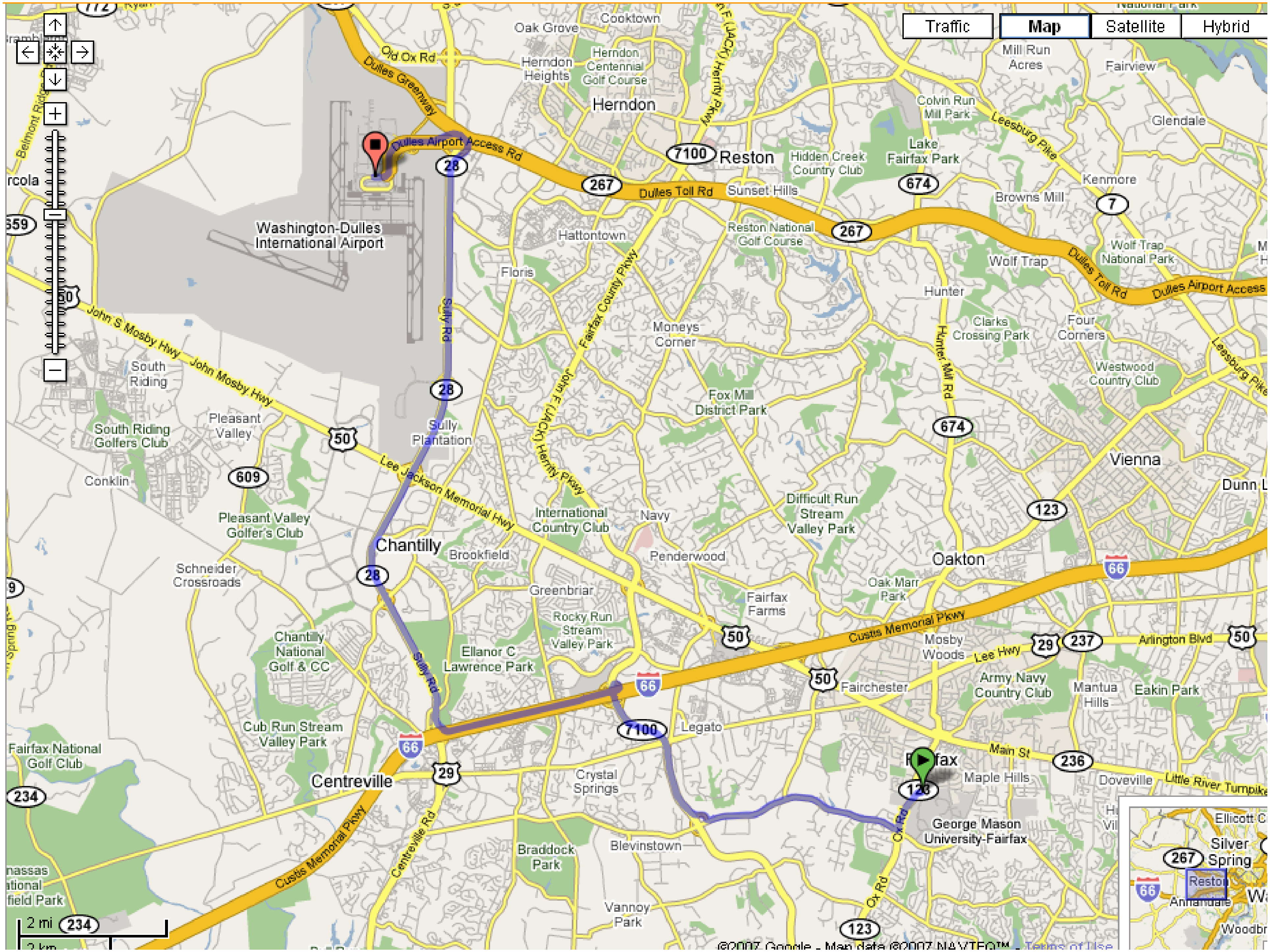
Example3 – Path Finding (Chap. 9)

- Problem: Find the optimal path from the origin to the destination subject to certain objectives
- *Input:*
  - A weighted graph
  - Origin and destination
- *Output:*
  - Optimal path



Traffic
Map
Satellite
Hybrid
Washington-Dulles International Airport
Dulles Airport Access Rd
Dulles Greenway
Old Ox Rd
Oak Grove
Cooktown
Herndon Heights
Herndon Centennial Golf Course
Herndon
Reston
Hidden Creek Country Club
Lake Fairfax Park
Mill Run Acres
Colvin Run Mill Park
Fairview
Glendale
Leesburg Pike
Kenmore
Browns Mill
Dulles Toll Rd
Sunset Hills
Reston National Golf Course
Hattontown
Floris
Fairfax County Pkwy
Wolf Trap
Wolf Trap National Park
Hunter
Clarks Crossing Park
Four Corners
Westwood Country Club
Moneys Corner
Fox Mill District Park
Hunter Mill Rd
Sully Rd
Sully Plantation
John S Mosby Hwy
John Mosby Hwy
South Riding
South Riding Golfers Club
Pleasant Valley
Conklin
Pleasant Valley Golfer's Club
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Chantilly
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Oakton
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Ellanor C Lawrence Park
Mosby Woods
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Arlington Blvd
Army Navy Country Club
Mantua Hills
Eakin Park
Fairchester
Legato
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Doveville
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Centreville Rd
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Blevinstown
Vannoy Park
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2 km
Silver Spring
Annandale
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Example4 – Interval Scheduling (Chap. 8 and 9)

- Problem: Maximize the ***maximum number or possible size*** of requests.
- *Input:*
  - A shared resource used by one person at one time
  - A bunch of requests
    - User i: Can I reserve the resource (classroom, book, supercomputer, microscope, ..) from time s_i to f_i?
- *Output:*
  - A selection of requests with assigned resource

Example5 – Stable Marriage (Chap. 10)

- A set of marriages is *stable* if there are no two people of opposite sex who would both rather have each other than their current partners.

- Problem: Find a stable marriage matching for given men and women to be paired off in marriages.

- *Input:*
  - n men and n women
  - Each person has ranked all members of the opposite sex with a unique number between 1 and *n* in order of preference
- *Output:*
  - A matching

Basic issues related to algorithms

- How to design algorithms
- How to express algorithms
- Proving correctness
- Efficiency
  - Theoretical analysis
  - Empirical analysis
- Optimality and improvement

Greatest Common Divisor Problem

- ***Problem:*** Find gcd(m,n), the greatest common divisor of two nonnegative, not both zero integers m and n
- ***Examples:*** gcd(60,24) = 12, gcd(60,0) = 60

Solution 1

- Observation: gcd(m,n) ≤ min{m,n}
- *Consecutive integer checking algorithm*
  - Step 1 Assign the value of min{*m,n*} to *t*
  - Step 2 Divide *m* by *t.* If the remainder is 0, go to Step 3; otherwise, go to Step 4
  - Step 3 Divide *n* by *t.* If the remainder is 0, return *t* and stop; otherwise, go to Step 4
  - Step 4 Decrease *t* by 1 and go to Step 2

Solution 2

- *Middle-school procedure*
  - Step 1 Find the prime factorization of *m*
  - Step 2 Find the prime factorization of *n*
  - Step 3 Find all the common prime factors
  - Step 4 Compute the product of all the common prime factors and return it as gcd(*m*,*n*)
- Example: gcd(60,24)
  - m = 60 = **2** x **2** x **3** x 5
  - n = 24 = **2** x **2** x 2 x **3**
  - gcd(m, n) = gcd(60,24) = **2** x **2** x **3** = 12
- Not an algorithm! *Prime factorization*

Prime Factorization

- *Input:* Integer $n \geq 2$
- *Output:* A sequence of prime numbers S, whose multiplication is n.
- *Algorithm:*

```
find a list of prime numbers P that are smaller than n
i ← 2
while i < n do
      if n%i = 0
         then s ← i, n ← n/I
         else i ← next prime number
```

Sieve

- *Input:* Integer $n \geq 2$
- *Output:* List of primes less than or equal to n
- *Algorithm:*

```
for p ← 2 to n do  A[p] ← p
for p ← 2 to ⌊n⌋ do
      if A[p] ≠ 0  //p hasn't been previously eliminated from the list
        j ← p * p
        while j ≤ n  do
              A[j] ← 0  //mark element as eliminated
              j ← j + p
```

Sieve (cont.)

- Example

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	3		5		7		9		11		13		15		17		19	
2	**3**		5		7				11		13				17		19	
2	3		**5**		7				11		13				17		19	

Solution 3 - Euclid's Algorithm

- Euclid's algorithm is based on repeated application of equality

$$\gcd(m,n) = \gcd(n, m \bmod n)$$

  until the second number becomes 0, gcd(m, 0) = 0 .
- Example: gcd(60,24) = gcd(24,12) = gcd(12,0) = 12
- *Algorithm*

```
while n ≠ 0 do
    r ← m mod n
    m ← n
    n ← r
return m
```

Algorithm design techniques/strategies

- Brute force
- Divide and conquer
- Decrease and conquer
- Transform and conquer
- Space and time tradeoffs
- Greedy approach
- Dynamic programming
- Iterative improvement
- Backtracking
- Branch and bound

Analysis of algorithms

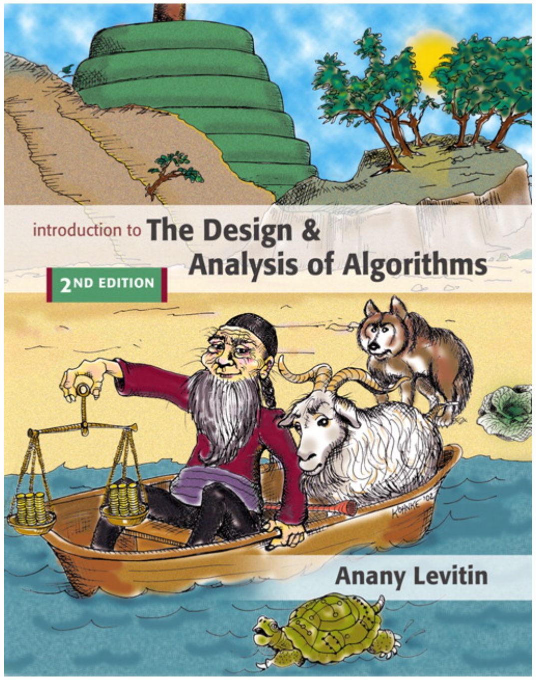
- How good is the algorithm?
  - time efficiency
  - space efficiency
- Does there exist a better algorithm?
  - Simplicity
  - Generality
  - lower bounds
  - optimality

Syllabus

- Lecture time
  - Tue & Thu 3:00-4:15pm
- Office Hour
  - Tue & Thu 4:30-5:30pm
  - Office: 443 ST II
- Course webpage:
  - www.cs.gmu.edu/~lifei/teaching/cs483_fall07/

Syllabus (cont.)

- TA: Yanyan Lu
  - Email: ylu4@gmu.edu
  - Office hour: Wed & Friday 4:00pm – 5:00pm
  - Room 437 STII
- Required Textbook:
  - ***Introduction to the Design and Analysis of Algorithms*** by Anany Levitin , Addison Wesley; 2nd edition (2007)



Syllabus (cont.)

- Topics
  - Analysis of Algorithm Efficiency
  - Brute Force
  - Divide (decrease) and Conquer
  - Transform and Conquer
  - Greedy Techniques
  - Dynamic Programming
  - Iterative Improvement
  - Limitations of Algorithm Power and Coping with Limitations

Syllabus (cont.)

- Grading (tentative)
  - Biweekly assignment (40%)
    - Work on your assignments independently.
    - List all the resources such as web, books and other students that may have helped with your solution.
    - Hand in hard copies.
    - One late submission (up to one week past the due date) per person per semester is permitted.
  - Midterm exam (25%)
  - Final exam (35%)
    - Two pages (letter size) of notes are allowed for both exams.

Some Suggestions

- Start working on assignments early
- Review notes and textbook after class
- Ask questions!

- Before next class
  - Read Chapter 1.1, 1.2, 1.4 and Appendix A.
- Next class
  - Algorithm analysis
  - Recursion