CS 310: Memory Hierarchy and B-Trees

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Week 14-1

Matrix Sum

Given an M by N matrix `X`, sum its elements

- M rows, N columns

Sum R

```
given X, M, N
sum = 0
for i=0 to M-1{
  for j=0 to N-1 {
    sum += X[i][j]
  }
}
```

Sum C

```
given X, M, N
sum = 0
for j=0 to N-1{
  for i=0 to M-1 {
    sum += X[i][j]
  }
}
```

- What's the difference?
- What's the complexity of each?
- Should the execution speed be different?

How does a CPU work?

CPU: Sees a load instruction

```
500:    lw   $t1, $t4
504:    lw   $t2, 4($t4)
508:    add  $t3, $t1, $t2
```

Load a *word* of memory

- Load value at address in register `t4` into register `t1`
- ex: `t4` contains the memory address 1024, integer 7 is there

Client/Server model

- CPU: requester
- Memory subsystem: provider
- Like you asking for a specific web page
  - Just viewed it a minute ago (fast)
  - GMU web site (medium)
  - Philippines hosted site (slow)

NUMA

When analyzing code, usually assume *uniform memory access*

- Same time to move any byte/word to a CPU register

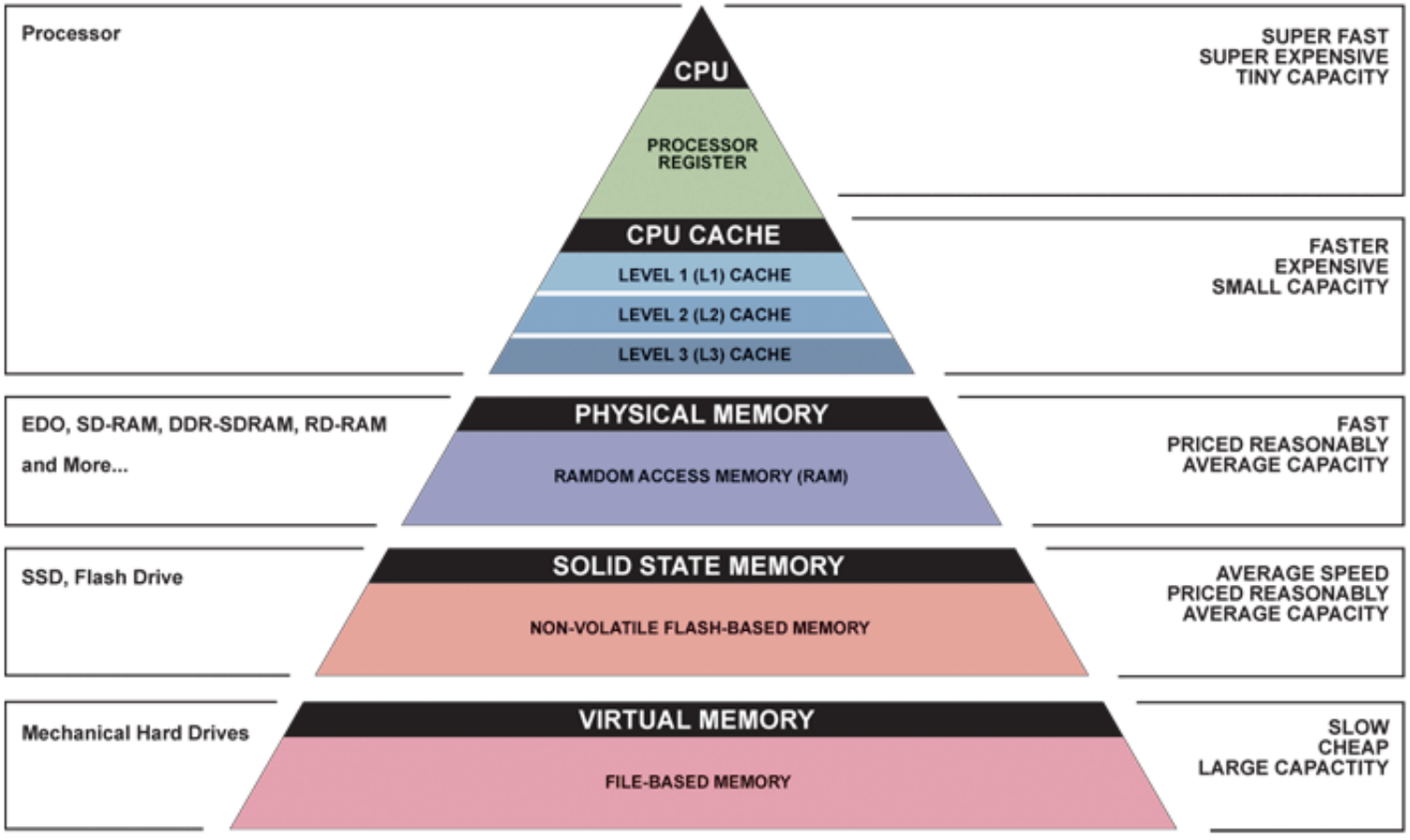
Real world: *non-uniform memory access*

- Some memory locations are "farther" away

The memory hierarchy

- Presents a uniform memory access interface
- Tries hard to provide it
- Fails

The Memory Pyramid



▲ Simplified Computer Memory Hierarchy
Illustration: Ryan J. Leng

Source Article

Numbers Everyone Should Know

Edited Excerpt of Jeff Dean's talk on data centers.

Reference	Time	Analogy
Register	-	Your brain
L1 cache reference	0.5 ns	Your desk
L2 cache reference	7 ns	Neighbor's Desk
Main memory reference	100 ns	This Room
Disk seek	10,000,000 ns	Salt Lake City

Does Big-O analysis capture these effects?

What's a Cache

```
500:   lw   $t1, $t4
```

`t4` contains address 1024, `lw` moves word at 1024 into register `t1`
Side-effect

- Memory addresses "around" 1024 are loaded into cache
- Probably something like addresses 1024 to 2047 (1K) end up in L1 cache
- Referred to as a *cache line*
- Subsequent accesses to 1028, 1032, ... 2044 will happen fast

Cache is a limited resource

- Putting one line in cache overwrites another line
- Later load address 5120, 1024-2047 evicted from cache

Cache Affects Performance

As measured by hardware counters using linux's `perf` on

```
model name : Intel(R) Core(TM) i5-2400 CPU @ 3.10GHz
cache size : 6144 KB
```

with

```
> perf stat $opts java MatrixSums 8000 4000 row
> perf stat $opts java MatrixSums 8000 4000 col
```

Measurement	row	col
cycles	3,507,364,715	5,605,621,966
instructions	2,353,887,029	2,543,165,478
L1-dcache-loads	527,694,054	561,540,169
L1-dcache-load-misses	25,638,014	122,663,199
Runtime (seconds)	1.001	1.620

L1 data cache load misses

- Row: 25K/548K = 4% main memory access
- Col: 122/585K = 20% main memory access

Cache and Main Memory

Concern

Binary search trees don't focus on exploiting cache very much

- Left/Right in cache 1-7ns access time
- Left/Right not in cache, 100ns trip to main memory
- Could do 100 operations during that trip (!)

Problem

- Left/Right not in main memory, 10,000,000 ns trip to disk
  - CPU gets a siesta, user gets irate
- When would this happen?

Big Data

- Machine named HAL has
  - 1mb cache
  - 8gb memory
- Database DB has
  - record size 2^{11} b (2048 bytes, 2kb)
  - 2^{24} records (16 mb)
  - Size: $2^{24} * 2^{11}$ b $= 2^{35}$ b (32gb)
- Find Record R in DB stored on HAL

Bad implementation

Store DB randomly, search sequentially for R

- Is the DB any bigger this way?

A bit better

Store DB as a balanced BST, binary searches for R

- How big is the new DB with left/right pointers?
- How deep is the tree?
- How many disk accesses may be needed?

Deep Trees

- Database DB has
  - record size 2^{11} b (2048 bytes, 2kb)
  - 2^{24} records (16 mb)
  - Size: $2^{24} \times 2^{11}$ b $= 2^{35}$ b (32gb)
- Find Record Z in Y stored on X
- Store DB in single BST, use binary search for R

Answers

- How big is the new DB with left/right pointers?
  - 2^{24} records
  - $2 \times$ 8b pointers per record for left/right = 16b per record $= 2^4$ b
  - $2^{24} \times 2^4 = 2^{28}$ b = 256mb
  - Small compared to 32gb (0.7%)
- How deep is the tree?
  - 2^{24} records, $\log_2$, expect 24 deep
- How many disk accesses may be needed?
  - Very unlucky - 24 accesses
  - Each costs 10,000,000 ns
  - Could have done 240,000,000 instructions

Tree + Array = B-Tree

Large DB's use sequential ordering with gaps, tree index

- Sequential chunks allow array-searching in cache
- Whole index doesn't fit in fast memory, but chunks do
- Do as much work as possible in fast memory to avoid slow disk access

B-trees exploit this to reduce tree depth / disk accesses

Internal Nodes

- Branch more than 2 ways
- Store multiple keys
- Keys in a sorted array
- Make sure they fit in cache
- Use a sequential search to find branch
- Always half full to full
  - root exception

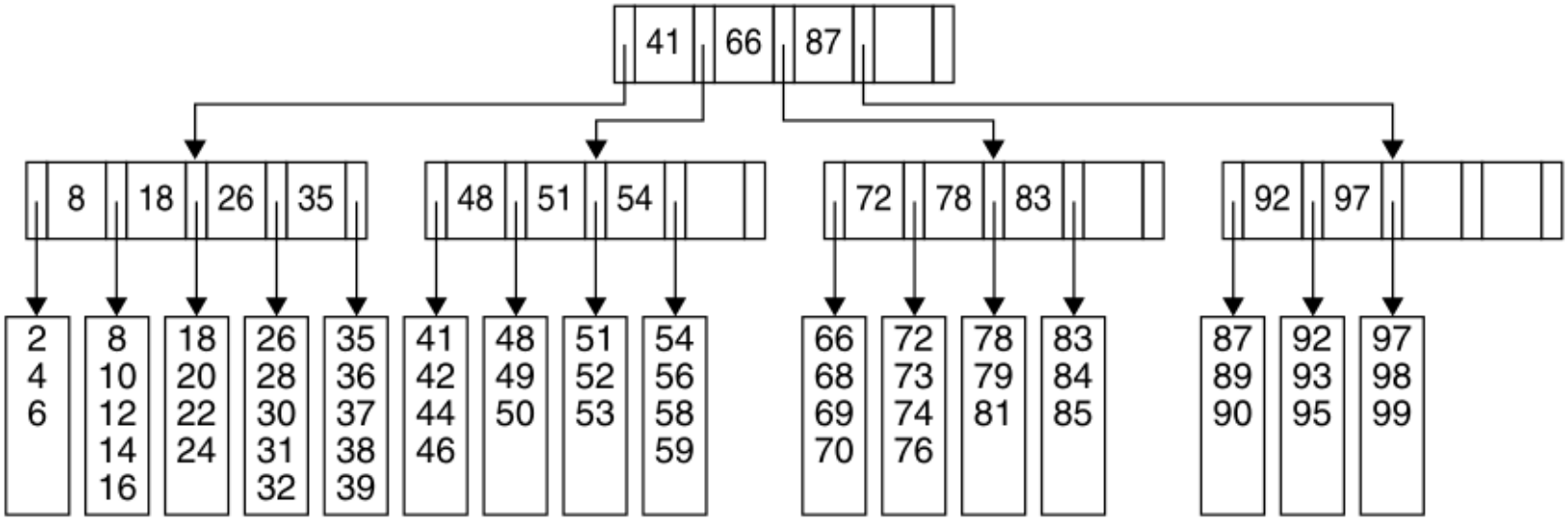
Leaves

- Data is only at the leaves
- Hold multiple sorted data
- Have maximum data capacity
- Optimized to disk *block size*
- Always half full to full

B-Trees

Weiss and Knuth: *Order 5* B-tree

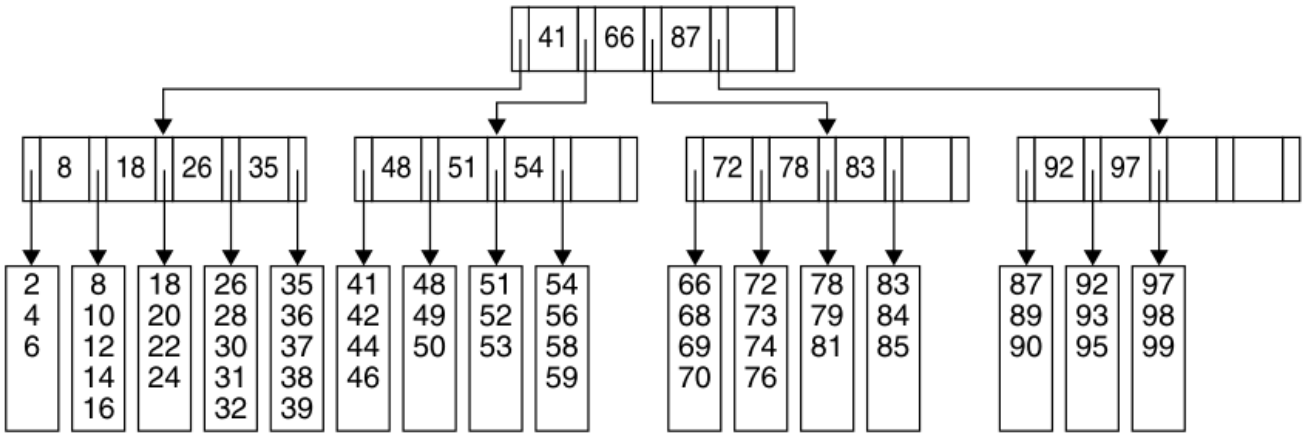
- Terminology is not standardized



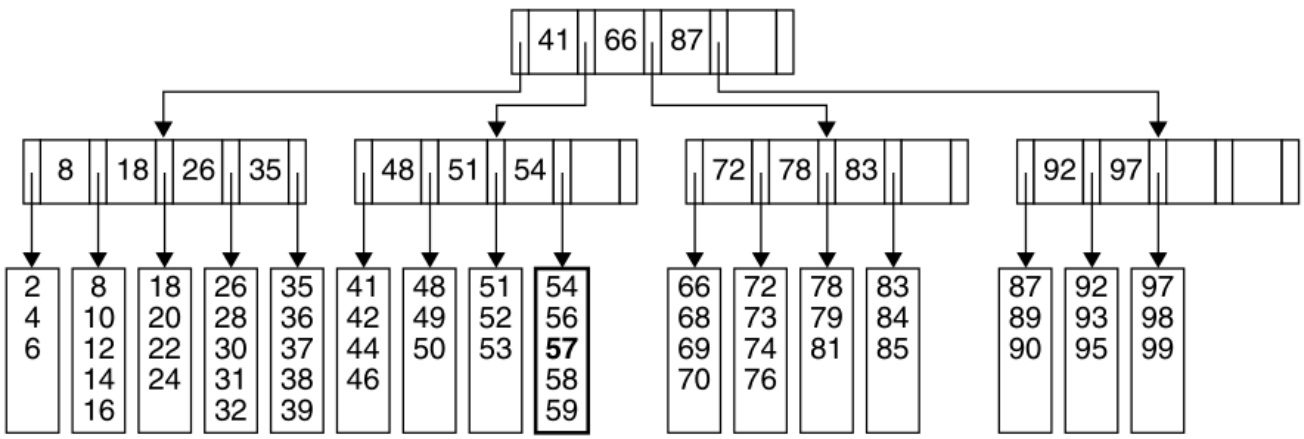
The origin of "B-tree" has never been explained by the authors. As we shall see, "balanced," "broad," or "bushy" might apply. Others suggest that the "B" stands for Boeing. Because of his contributions, however, it seems appropriate to think of B-trees as "Bayer"-trees.
– *Wikipedia: B-tree*

B-Trees Ops

Original



Insert 57



B-Trees Ops

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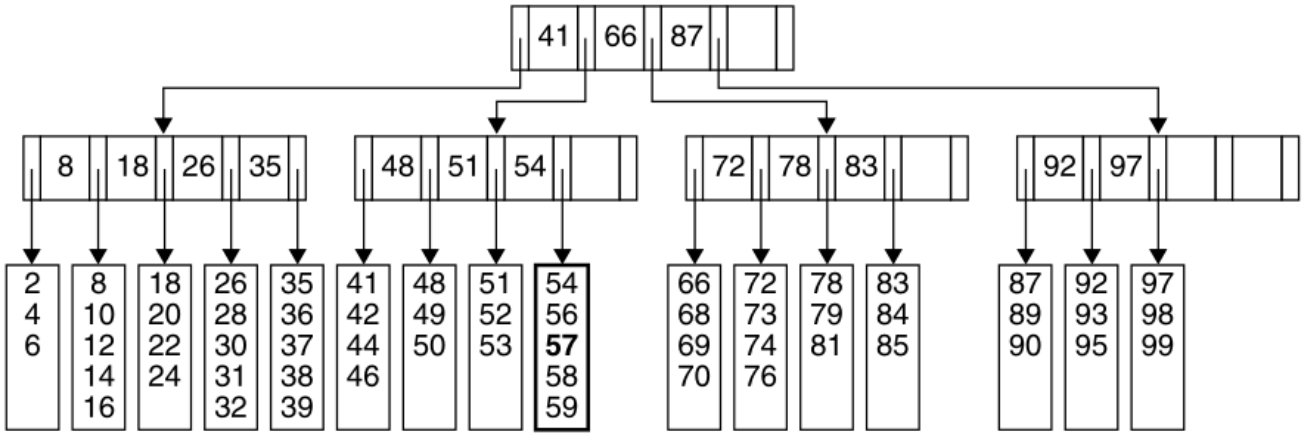
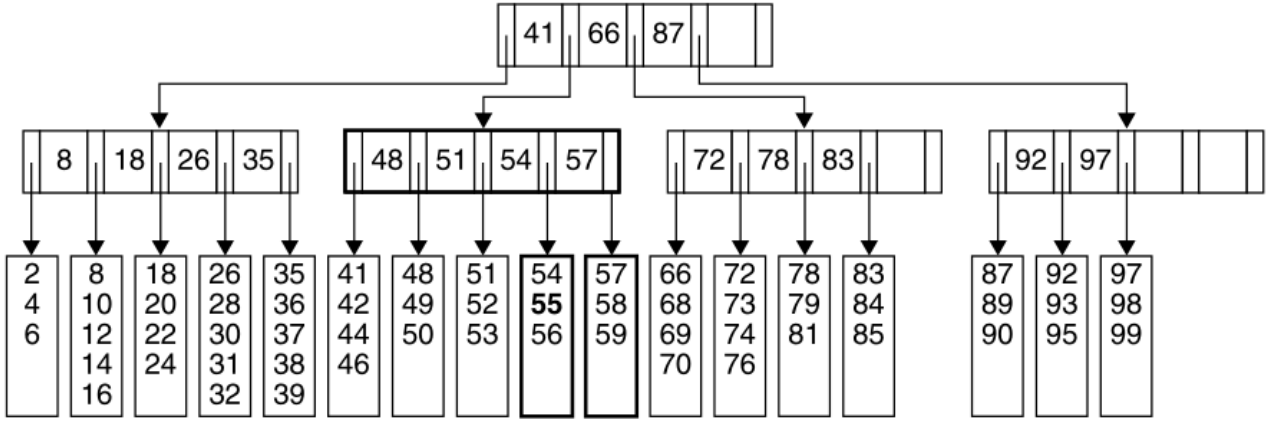
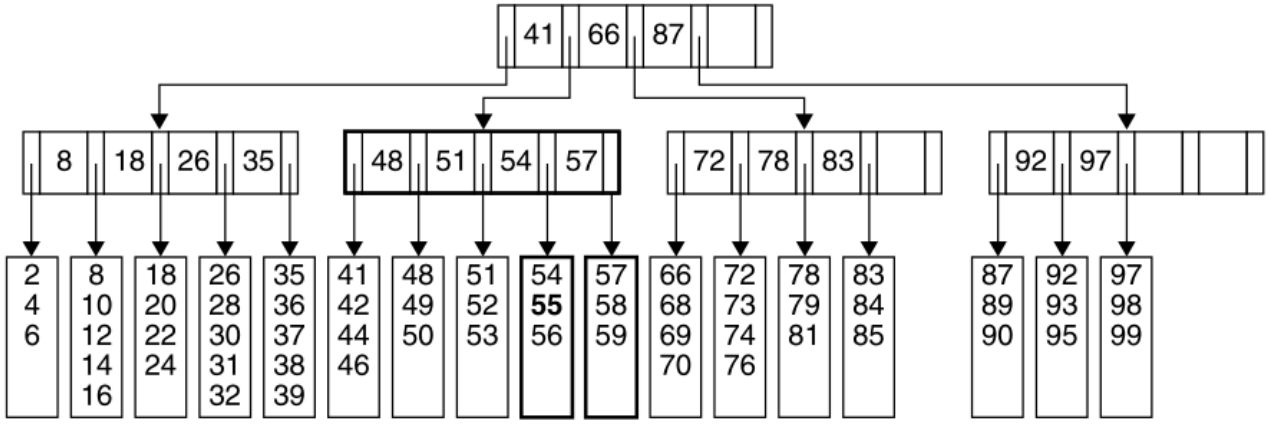


Insert 55

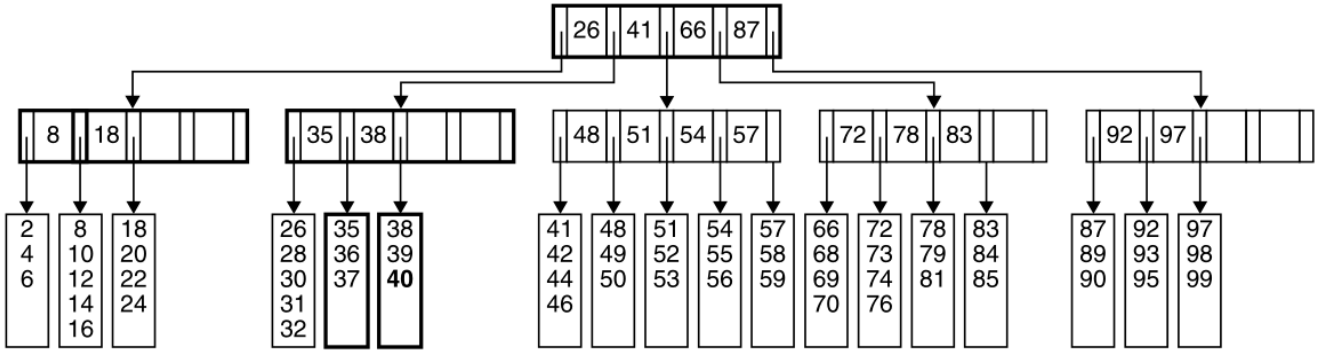


B-Trees Ops

Inserted 55

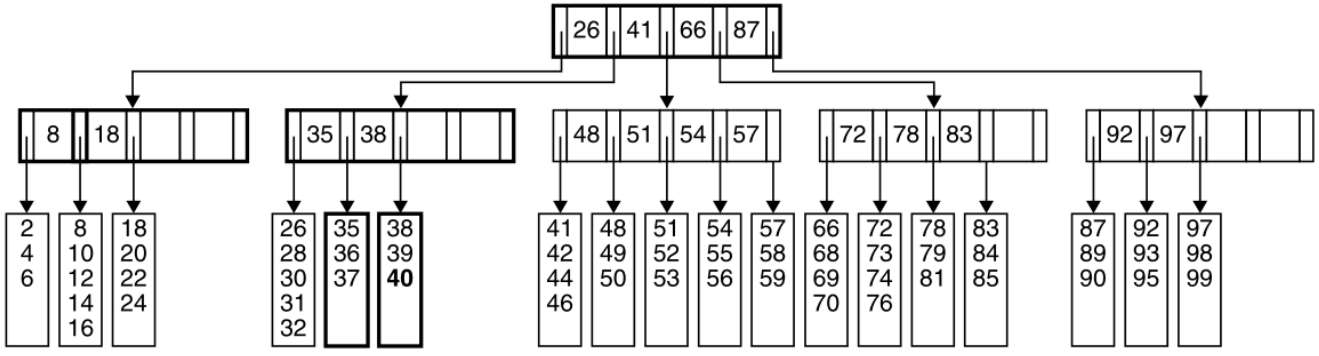


Insert 40

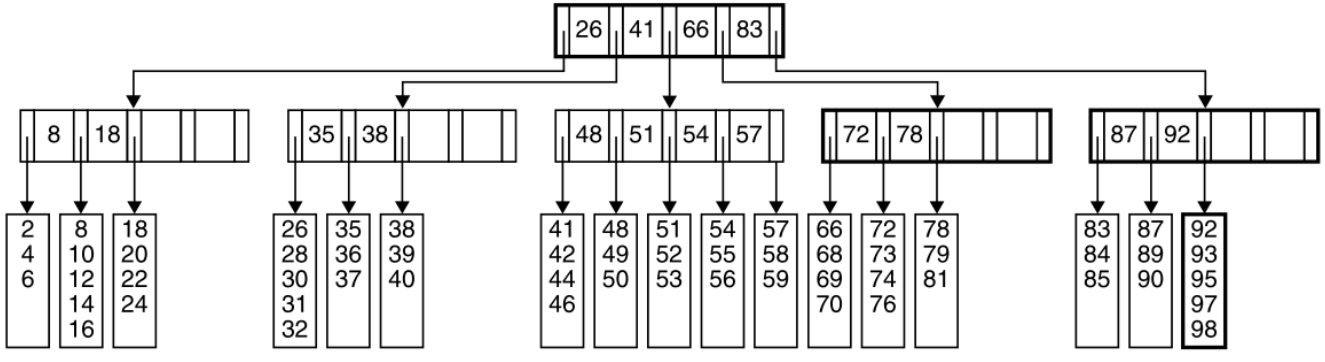


B-Trees Ops

Inserted 40



Delete 99



General Strategies

ADD() quasi-code

```
ADD(x,bt)
  find right leaf in bt
  if space in leaf
    add x to leaf
  else
    if parent has room
      new leaf
      split data
      add x to leaf
    else
      recurse up
      split internal
      new leaves
      split data
      back down to add x
```

REMOVE() quasi-code

```
REMOVE(x,bt)
  find leaf with x
  remove x

  if leaf < 1/2 full
    merge with neighbor leaf
    steal leaves if needed
    recurse up to adjust
```

B-tree Take-home

- Multi-way trees
- If order-k nodes are all 1/2 full $\rightarrow$ $O(\log_k N)$ height
- Hybrid of array/tree
- Good for data that doesn't fit in memory
  - Large Databases
  - Filesystems
  - Sensitive to memory hierarchy
- Simple idea, complex implementation
- Many variations on the idea
- No Weiss B-trees: too complex