# CS 310: Binary Search Trees

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Week 11-1

# Logistics

## Reading

- Weiss Ch. 7 Recursion
- Weiss Ch. 19 BSTs

## Today

- Tree traversals
- Mention Tree Iterators
- Comparable vs Comparator
- Binary Search Trees

## HW2

- Due Thursday
- Questions?

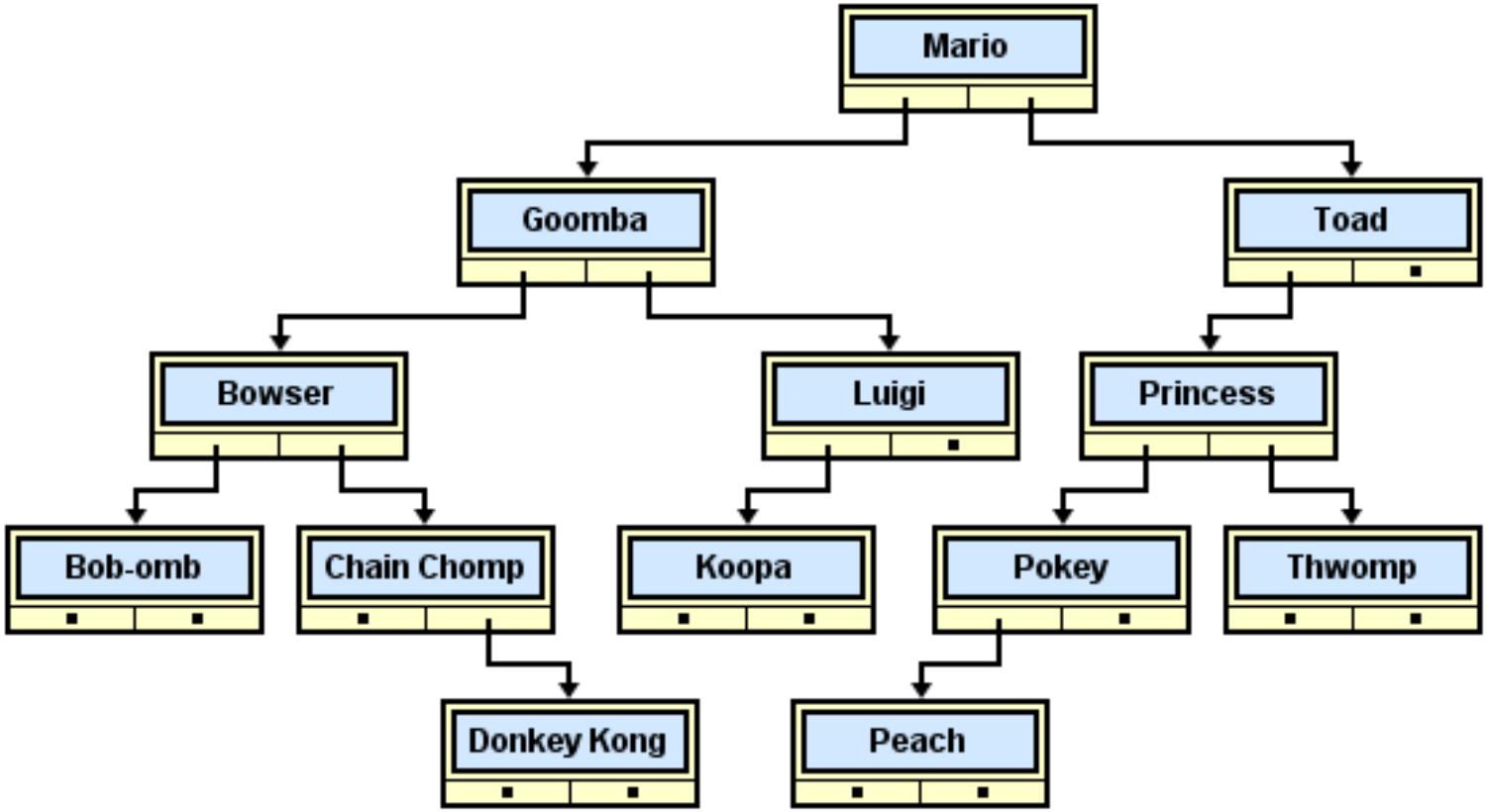
## HW3

- Will post and discuss on Thursday
- Last HW of the course

# Binary Search Tree Property

A binary tree where every node `N` in the BST

- Any data in the tree rooted at `N.left` sort before `N.data`
- Any data in the tree rooted at `N.right` sort after `N.data`



# Comparisons

How does java guarantee comparability?

### Comparable

Data can implement Comparable

```
int c = x.compareTo(y);
// neg for x before y
//   0 for x equals y
// pos for x after  y
```

### Comparator

Use a Comparator object to do comparisons

```
Comparator<Thing> cmp =
   new ...;
int c = cmp.compare(x,y);
// neg for x before y
//   0 for x equals y
// pos for x after  y
```

- Generalizes subtraction: `c = x-y;`
- Presence of both hints at a fundamental problem

# Comparators allow multiple sorting criteria

```
public class RCComp<RowCol> {
  public int compare(RowCol a,
                     RowCol b)
  {
    // compare a.row to b.row
    // if tied, compare a.col to b.col
  }
}

public class CRComp<RowCol> {
  public int compare(RowCol a,
                     RowCol b)
  {
    // compare a.col to b.col
    // if tied, compare a.row to b.row
  }
}
```

```
main(){
  RowCol x = ...;
  RowCol y = ...;
  Comparator<RowCol> rc = new RCComp();
  if(rc.compare(x,y) < 0){
    // x before y according to rc
  }
  else if(rc.compare(x,y) > 0){
    // x after y according to rc
  }
  else{
    // x equals y according to rc
  }

  Comparator<RowCol> cr = new CRComp();
  if(cr.compare(x,y) < 0){
    // x before y according to cr
  }
  else if(cr.compare(x,y) > 0){
    // x after y according to cr
  }
  else{
    // x equals y according to cr
  }
}
```

# Comparison Type Conventions

`java.util.TreeSet` and similar trees EITHER

1. Contain objects that implement `Comparable` (things in tree can compare to one another via `x.compareTo(y)`
2. Constructed with a `Comparator` as in

```
Comparator<Something> cmp = ...;
TreeSet<Something> t = new TreeSet<Something>(cmp);
```

In example code, we'll presume objects are `Comparable` for convenience

## Generics get Funky

```
class Tree< T extends Comparable<? super T> >
```

Tree of something stuff that *descends from something Comparable to T*

```
class Tree< T extends Comparable<T> >
```

Tree of stuff that is comparable to itself only

# Define `bst.find()`

- `find(T x)` is publicly accessible

  `tree.find("Mario");`

- Define

  `find(T x, Node<T> t)`

  which works on a given start node

- Compare via `Comparable`:

  `if(x.compareTo(t.data) < 0)`

Give 2 versions

- Recursive
- Iterative

```
public class BinarySearchTree
  <T extends Comparable<T>>
{
  protected Node<T> root;
  // Return x if in tree,
  // null otherwise
  public T find( T x ){
    Node<T> result =
      find(x, this.root);
    if(result == null){ return null;}
    else{ return result.data; }
  }

  // Find node containing x
  // starting at node t
  // Return null if not found
  private static
  Node<T> find(T x, Node<T> t){
    // DEFINE ME
  }
}
```

# Recursive `find(x,node)`

Use key of data to search through tree

- Left for less than
- Right for greater than

```
// pseudocode
Node<T> find(T x, Node<T> t){
  if(t == null){
    return null;
  }
  int diff = x.compareTo(t.data);
  if(diff < 0){                    // x < t.data
    return find(x,t.left);
  } else if(diff > 0){             // x > t.data
    return find(x,t.right);
  } else {                         // x==t.data
    return t.data;                 // found
  }
}
```

# Iterative find(x,node)

See weiss/nonstandard/BinarySearchTree.java

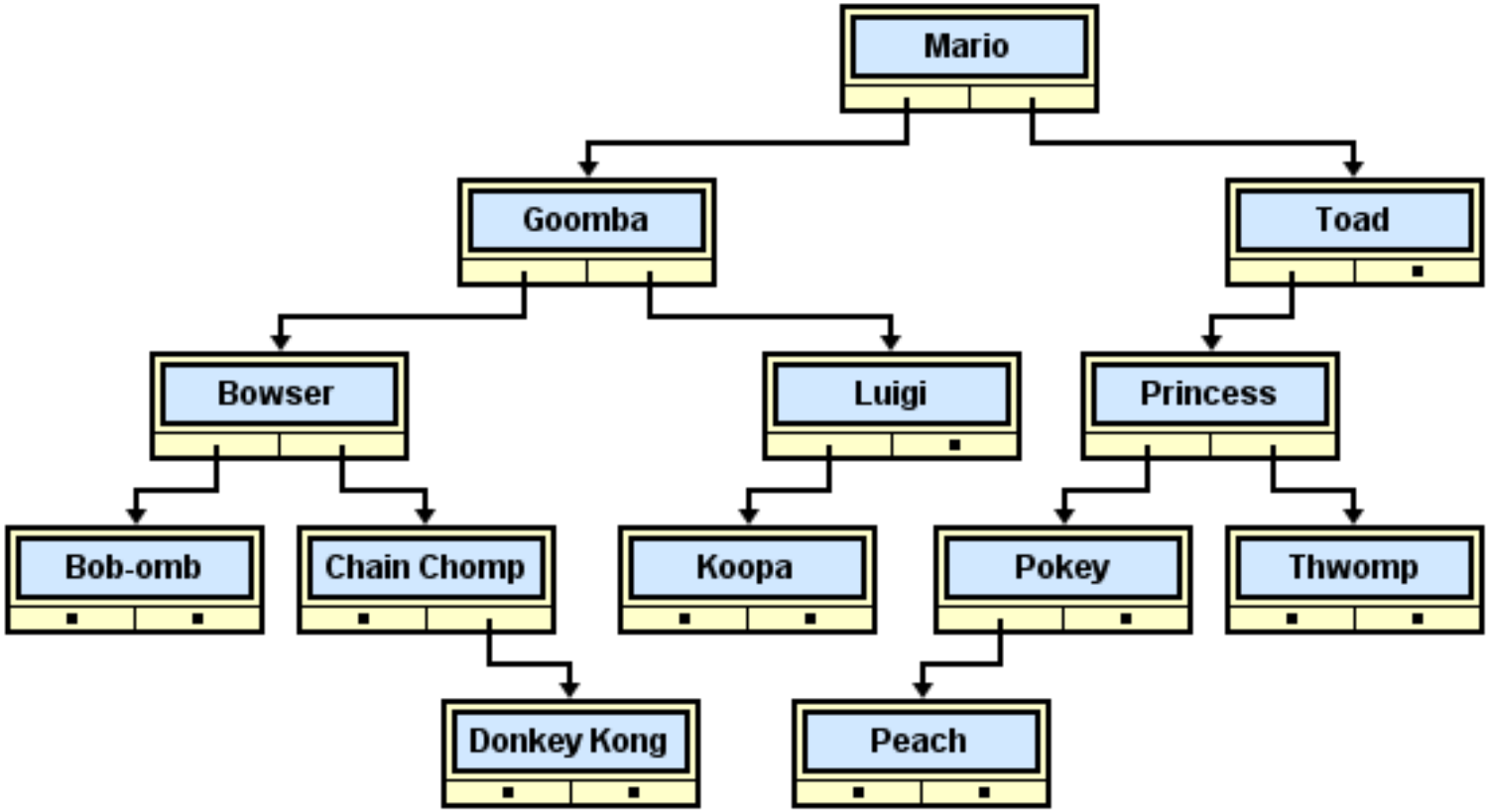
```
private static BinaryNode<T> find(T x, BinaryNode<T> t){
  while( t != null ) {
    if( x.compareTo( t.data ) < 0 )
      t = t.left;
    else if( x.compareTo( t.data ) > 0 )
      t = t.right;
    else
      return t;                    // Match
  }
  return null;                     // Not found
}
```

# Complexity of BST Find

- What is the worst-case complexity of `find(x)` in terms of tree properties?
- Construct a tree with this this worst case complexity

# Examples of Insert

Play with `MyBST.java` in JGrasp and look at the pretty pictures after multiple `insert(x)` calls



# Insertion: Similar to `find(x)`

- May need to change a `left` or `right` pointers, redefine root
- No duplication, define a `TreeSet`, exception on duplicate insert

Define Recursive Insert

```
class BinarySearchTree<T> {
  Node<T> root=null; int size=0;
  public void insert( T x ){
    root = insert( x, root );
    this.size++;
  }
  private static Node<T> insert( T x, Node<T> t ){
    // DEFINE ME
  }
}
```

Define Iterative Insert if You're Brave

```
public void insert( T x ){
  // DEFINE ME
}
```

# Exercise: Recursive insert(x,t)

From weiss/nonstandard/BinarySearchTree.java

```
class BinarySearchTree<T> {
  Node<T> root;
  public void insert( T x ){
    root = insert( x, root );
  }
  private static Node<T> insert( T x, Node<T> t )
  {
    if( t == null )
      t = new Node<T>( x );
    else if( x.compareTo( t.data ) < 0 )
      t.left = insert( x, t.left );
    else if( x.compareTo( t.data ) > 0 )
      t.right = insert( x, t.right );
    else
      throw new DuplicateItemException( x.toString( ) );
    return t;
  }
}
```

# Compare: Iterative insert(x,t)

```
class BinarySearchTree<T> {
  Node<T> root;
  public void insert( T x ){
    if(this.root == null){            // New root?
      this.root = new Node<T>(x);     // Yes
      return;                         //
    }                                 //
    Node<T> t = this.root;            // Root exists
    while(true){                      // Traverse tree
      int diff = x.compareTo(t.data); //
      if(diff == 0){                  // Check duplicate
        throw new DuplicateItemException( x.toString( ) );
      }
      if(diff < 0){                 // Go left
        if(t.left == null){         // Found insertion point?
          t.left = new Node<T>(x); // Yes
          return;                   //
        }                           //
        else{                       // No: Go farther left
          t = t.left;
        }
      }
      else{                          // Go right
        if(t.right == null){         // Found insertion point?
          t.right = new Node<T>(x); // Yes
          return;                    //
        }                            //
        else{                        // No: Go farther left
          t = t.right;
        }
      }
    }
  }
}
```

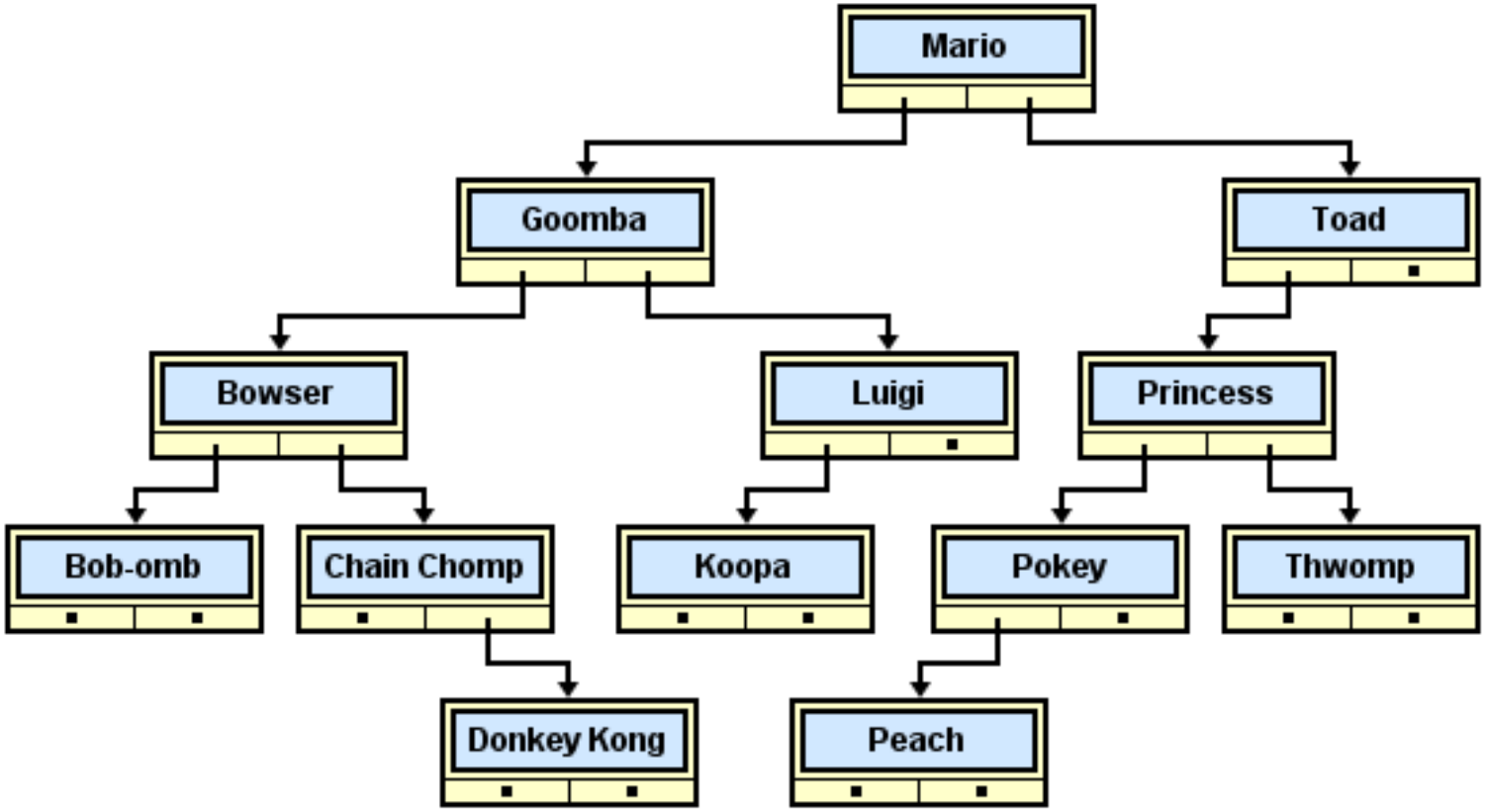
# Binary Search Tree `remove(x)`

```
// Public method, eliminate x if present in tree
public void remove( T x );

// Recursive helper method
private Node<T> remove( T x, Node<T> t );
```

- Get rid of a node with data `x` in a binary tree; throw exception if not present (or ignore request)
- More involved than `find/insert`
- Preserve Tree Structure
- Recursion greatly eases implementation

# Prelims

Consider Mario Tree




- Describe which cases exist `tree.remove(x)`?
- Which of these do you anticipate being easy/hard to code for?

# BST General `remove(x)`: Cases

Cases for `t.remove(x)`

1. `x` not in tree
   - Leave tree as is or raise an exception
2. `x` at a node with no children
   - Get rid of node containing `x`
3. `x` at a node with 1 child
   - "Pass over" node containing `x`
4. `x` at a node with 2 children
   - Find a next node in sorting order
   - Replace `x` with next nodes data
   - Remove next node
   - Next is minimum of right subtree

# Exercise: Useful Helper Methods

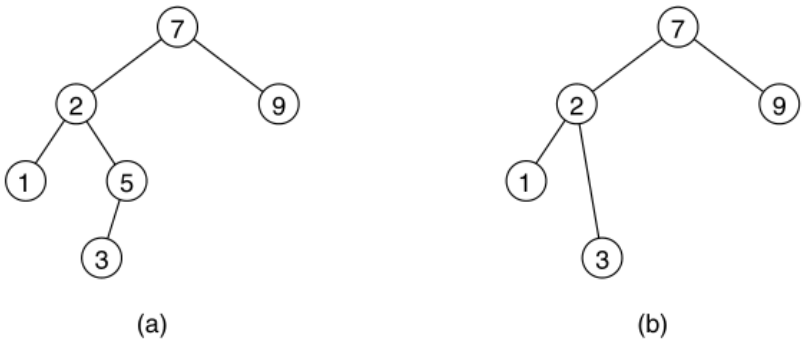
```
class BST<T> {
  private Node<T> root;

  // Public facing method, find minimum element and return it
  public T findMin(){  return this.findMin(this.root); }

  // Private helper method return the smallest element in the
  // tree rooted at t
  private T findMin(Node<T> t){
    // DEFINE ME
  }

  // Public facing method, eliminate the smallest data in tree
  public void removeMin(){  this.root = removeMin(this.root); }

  // Recursive helper; remove the node with the smallest data
  // in it in the tree rooted at t.  The node returned is used
  // to alter the structure of the tree.
  private Node<T> removeMin(Node<T> t){
    // DEFINE ME
  }
}
```

# Warm-up Questions

1. What is the Binary Search Tree property?
2. Are all trees binary trees? Do all binary trees have the BST property? (give counter-examples)
3. Where is the biggest data element in a BST? The smallest?
4. What are the runtime complexities of BST `tree.find(x)` and `tree.insert(x)`?
5. Which kinds of nodes are easy to remove from BSTs? Which kinds are more difficult?
6. What is a useful strategy for removing difficult nodes?

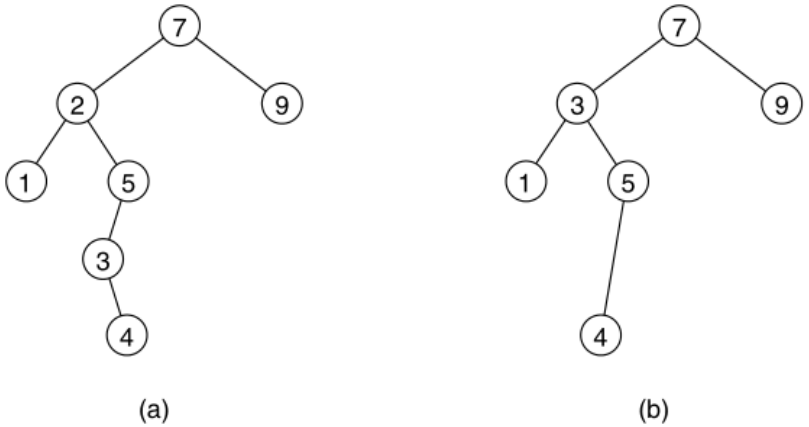
# Children Cases for `remove(t,x)`

## One Child: Remove 5




1. Find node `t` with data `x`
2. Replace with only child

## Two Children: Remove 2




1. Find node `t` with data `x`
2. Find `min` node of `t.right`: `min` must have 0/1 child
3. Replace `t.data` with `min.data`
4. Remove `min`

# Recursive Implementation: Think Locally

## Lesson from `insert()`

- Recall in `insert(x,t)`, did stuff like

```
t.right = insert(x, t.right);
// a new/existing node is returned by insert()
```

- Take the same approach for `remove(x,t)`
- Assume these helpers are Available

```
T findMin(Node<T> t);   Node<T> removeMin(Node<T> t)
```

## Implement Recursive `remove(x,t)`

- How to know if `t` is *the* node?
- What to do if `t` isn't *the* node?
- If `t` is *the* node, are there separte cases for action?

# Cases for recursive `remove()`

1. ☒ `t` is `null`

   Throw an exception

   ```
   throw new ItemNotFoundException();
   ```

   Or do nothing to the tree

   ```
   return null;
   ```

2. ☒ `x` less than `t.data` (recurse left)

   ```
   t.left = remove(t.left, x);
   ```

3. ☒ `x` greater than `t.data` (recurse right)

   ```
   t.right = remove(t.right, x);
   ```

4. ☐ `x` equals `t.data` (remove `t`)
   - `t` has 0 children, get rid of `t`
   - `t` has 1 child, pass over `t`
   - `t` has 2 children, replace with next/prev

# Case 4: x equals t.data (remove t)

Helper methods defined elsewhere

```
T findMin(Node<T> t);   Node<T> removeMin(Node<T> t)
```

- t has 0 children, get rid of t

  ```
  return null;
  ```

- t has 1 child, pass over t

  ```
  (t.left!=null) ? return t.left : return t.right;
  ```

- t has 2 children, replace with next or prev

  ```
  t.data = findMin(t.right);
  t.right = removeMin(t.right);
  return t;
  ```

- How are `findMin(t)` and `removeMin(t)` implemented?
  - Where is the minimum node in a tree?
  - How many children does it have?

# remove(x)

Adapted from weiss/nonstandard/BinarySearchTree.java

```
private Node<T> remove( T x, Node<T> t ){
  if( t == null )
    throw new ItemNotFoundException( x.toString( ) );
  if( x.compareTo( t.data ) < 0 )
    t.left = remove( x, t.left );
  else if( x.compareTo( t.data ) > 0 )
    t.right = remove( x, t.right );
  // Found at this node
  else if( t.left != null && t.right != null ){
    // Two children
    t.data = findMin( t.right );
    t.right = removeMin( t.right );
  }
  else
    // One child or no children
    t = ( t.left != null ) ? t.left : t.right;
  return t;
}
```

# So Far

## Binary Search Trees

- Defined `find()` / `insert()` / `remove()`
- Helpers: `findMin()` / `findMax()` / `removeMin()` / `removeMax()`
- All ops runtime complexity $O(Height)$
- Discuss balancing trees to ensure that $Height \approx \log(Size)$

## Next Time

- Balanced BSTs: AVL and Red-Black Trees
- Reading: Weiss Ch. 19.4-5