CS 310: Hash Table Collision Resolution

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Week 7-1

Logistics

Reading

- Weiss Ch 20: Hash Table
- Weiss Ch 6.7-8: Maps/Sets

Goals Today

- Hash Functions
- Separate Chaining In Hash Tables

HW 2

- Milestones due Thursday
- Final tests up by Friday

Upcoming

Mon	10/10	No class
Tue	10/11	Hash functions
		Hash Tables
Wed	10/12	Midterm review
Mon	10/17	Midterm Exam
Wed	10/19	Hash tables
		Maps & Sets

Midterm Next Week Topics to begin reviewing

- Big-O complexity for runtime and space/memory consumption, analyzing existing code to determine its properties
- ArrayLists, LinkedLists (single and doubly linked), Stacks/Queues implemented with arrays/linked nodes, iterators, hash functions, hash tables (separate chaining)
- Appropriate use of these data structures for application settings
- Java generics for data structures

Hash Table Class So Far...

So far

- Know: how to use `int xhc = x.hashCode();`
- Simple Hash Set with `add(x)/contains(x)` has an array `hta`
- Put `x` in `hta[]` based on `xhc`

Answer

- What if `xhc` is out of bounds in `hta`?
- Unconditionally set `hta[xhc]` to `x` in `add(x)`?

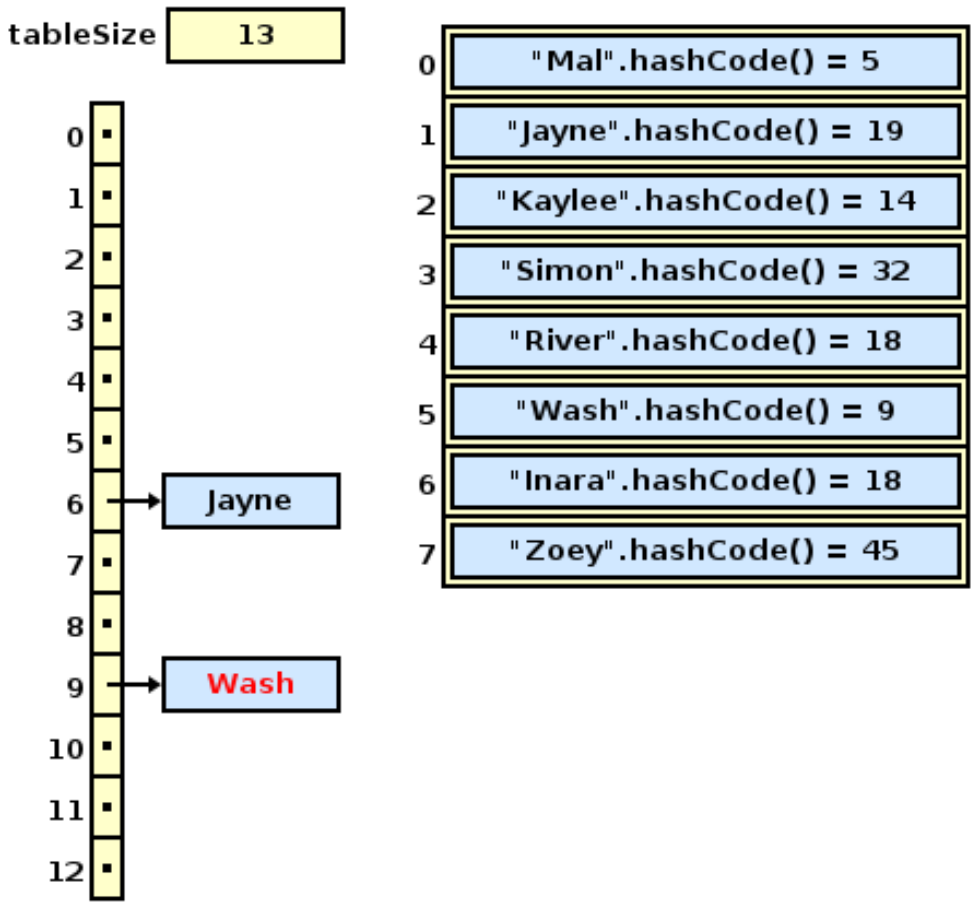
```
class MyHashSet<T>{
  T hta[];  int size;
  boolean contains(T x){
    int xhc = x.hashCode();
    // If xhc out of bounds?
    xhc = ???;
    // Is this okay?
    return
      x.equals(this.hta[xhc]);
  }
  void add(T x){
    int xhc = x.hashCode();
    // If xhc out of bounds?
    xhc = ???;
    // Is this okay?
    this.hta[xhc] = x;
    this.size++;
  }
}
```

Getting Hash Codes in Bounds

- `hta[]` has a fixed size
- The hash code `xhc` can be any integer
- Take an absolute value of `xhc` if negative
- Use modulo to get `xhc` in bounds

```
int n = hta.length;
hta[abs(xhc) % n] = x;
```

Note: For mathy reasons we'll briefly discuss, usually make hash table size `n` a *prime number*



Pragmatic Collision Resolution: Separate Chaining

Motivation

- Put `x` in table at `hta[xhc]`
- Problem: What if `hta[xhc]` is occupied?

Separate Chaining

Most of you recognize this problem can be solved simply

- Internal array contains lists
- Add `x` to the list at `hta[xhc]`

```
public class HashTable<T>{
  private List<T> hta[];
  ...
```

Separate Chaining: Example

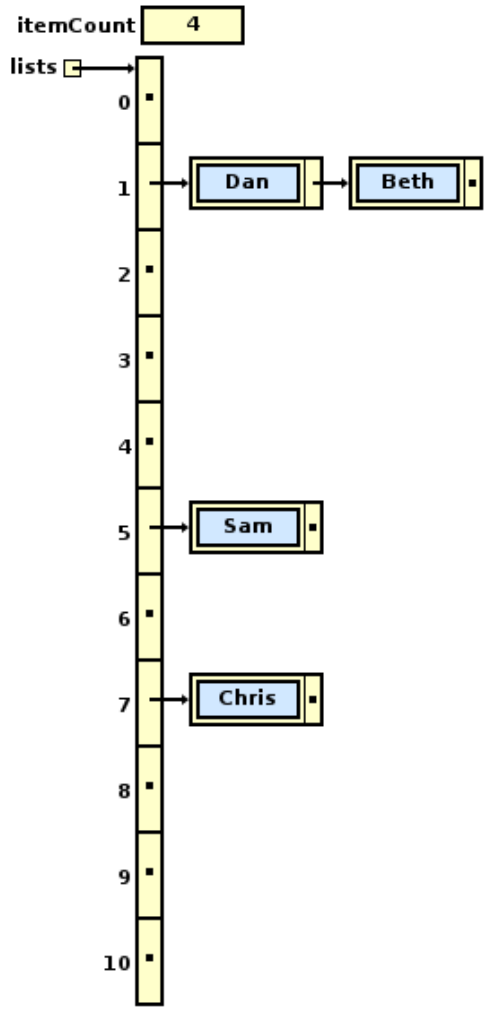
Code

```
String [] sa1 = new String[]{
  "Chris","Sam","Beth","Dan"
};

SeparateChainHS<String> h =
  new SeparateChainHS<String>(11)

for(String s : sa1){
  h.add(s);
}
print(h.load());
// load = 4 / 11
// 0.3636363636363636 5
```

Load = 0.36



$$load = \frac{\text{item count}}{\text{array length}}$$

Separate Chaining: Example

Code

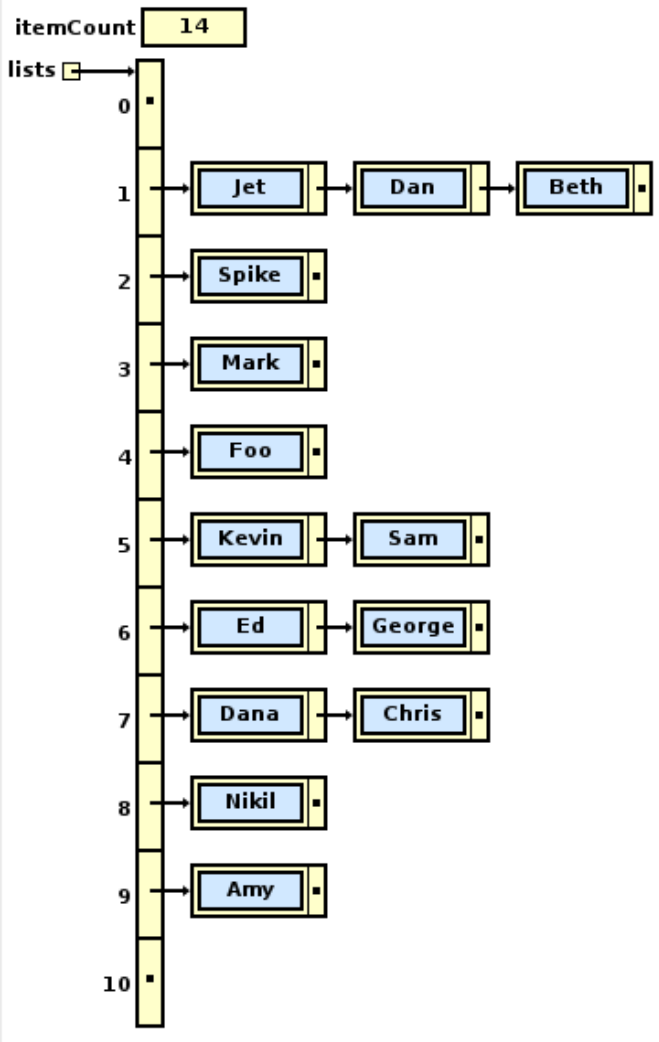
```
String [] sa2 = new String[]{
  "Chris","Sam","Beth","Dan",
  "George","Kevin","Nikil",
  "Mark","Dana","Amy","Foo",
  "Spike","Jet","Ed"
};

SeparateChainHS<String> h =
  new SeparateChainHS<String>(11)

for(String s : sa2){
  h.add(s);
}

h.load();
// load = 14 / 11
// 1.2727272727272727
```

Load = 1.27



Implement Separate Chaining

- A Set has at most one copy of any element (no duplicates)
- Write add/remove/contains for SeparateChainingHS
- What are the time complexities of each method?

```
public class SeparateChainingHS<T>{
  private List<T> hta[];
  private int itemCount;

  // Constructor, n is initial size of hta[]
  public SeparateChainingHS(int n){
    this.itemCount = 0;
    this.hta = new List<T>[n];
    for(int i=0; i<n; i++){
      this.hta[i]=new LinkedList<T>();
    }
  }

  public void add(T x);          // Add x if not alread present
  public void remove(T x);       // Remove x if present
  public boolean contains(T x); // Return true if x present, false o/w
}
```

Separate Chaining Viable in Practice

Java's built-in hash tables use it

- Simple to code
- Reasonably efficient
- `java.util.HashSet / HashMap / Hashtable` all use separate chaining

Code shown in Weiss pg 799

- Rolled own linked list
- No `remove` (write it yourself)
- Part of code distribution

Analyses of methods are influenced by Load

$$load = \frac{\text{item count}}{\text{array length}}$$

Analysis

`add()`

`add(x)` is $O(1)$ assuming adding to a list is $O(1)$

```
int xhc = x.hashCode();
List l = hta[ abs(xhc) % hta.length];
l.add(x);
```

`remove()/contains()`

- Assume fair hash function (distributes well)
- Load is the average number of things in each list in the array.
- `remove(x)/contains(x)` must potentially look through *Load* elements to see if `x` is present
- Therefore complexity $O(Load) = O(itemCount/arraySize)$

Alternatives to Separate Chaining

Separate Chaining works well but has some disadvantages

- Requires separate data structure (lists)
- Involves additional level of indirection: elements are two or three additional memory references away from the hash table array
- Adding requires memory allocation for nodes/lists

Alternative: Open Address Hashing

- Ban the use of lists in the hash table
- Store element references *directly in hash table array*
- Why do it this way?
- How can we handle collisions now?

Open Addressing

Basic Design

- Hash table elements stored in array `hta` (no auxilliary lists)
- Probe a sequence of entries for object

```
# Generic pseudocode for a probe sequence
pos = abs(x.hashCode() % hta.length);
repeat
  if hta[pos] is empty
    hta[pos] = x
    return
  else
    pos = someplace else
```

Design Issues

- Obvious *next* places to look after `pos`?
- How to indicate an entry is empty?
- Limits?

Linear Probing

Start with normal insertion position `pos`

```
int pos = Math.abs(x.hashCode() % hta.length);
```

Try the following sequence until an empty array element is found

```
pos, pos+1, pos+2, pos+3, ... pos+i
```

Process of `add(x)` in hash table

```
// General idea of linear probing sequence
pos = Math.abs(x.hashCode() % hta.length);
if hta[pos] empty, put x there
else if hta[(pos+1)] empty, put x there
else if hta[(pos+2)] empty, put x there
...
```

Write java code for this

```
// Insert x using linear probe sequence
public void add(T x)
```

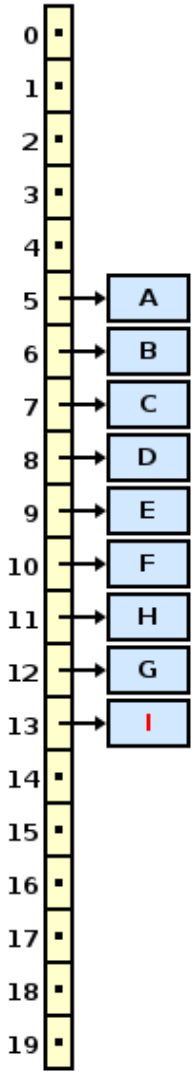
Consequences of Open Address Hashing

With linear probing

- Can `add(x)` fail? Under what conditions?
- Code for `contains(x)`?
- How does `remove(x)` work?

Removal in Open Addressing: Follow Chain

Item	Code	Pos	Added
A	5	5	1
B	6	6	2
C	5	7	3
D	7	8	4
E	5	9	5
F	8	10	6
G	11	11	7
H	12	12	8
I	9	13	9



- Suppose `remove(X)` sets position to `null`
- What are the booleans assigned to?

```
h.remove(A); boolean b1 = h.contains(C);
h.remove(D); boolean b2 = h.contains(F);
h.remove(E); boolean b3 = h.contains(I);
```

Avoid Breaking Chains in Removal

- Don't set removed records to `null`
- Use place-holders, in Weiss it's `HashSet.HashEntry`

```
private static class HashEntry {
  public Object  element;   // the element
  public boolean isActive;  // false if marked deleted
  public HashEntry( Object e ) {
    this( e, true );
  }
  public HashEntry( Object e, boolean i ){
    element  = e;
    isActive = i;
  }
}
```

Explore `weiss/code/HashSet.java`

- `remove(x)` sets `isActive` to `false`
- `contains(x)` treats slot as filled
- `rehash()` ignores inactive entries

Load and Linear Probing

Load has a big effect on performance in linear probing

- When Inserting `x`
- If `h[cx]` full, `cx++` and repeat
- When `h` is nearly full, scan most of array
- $load \approx 1 \rightarrow O(n)$ for `add(x)/contains(x)`

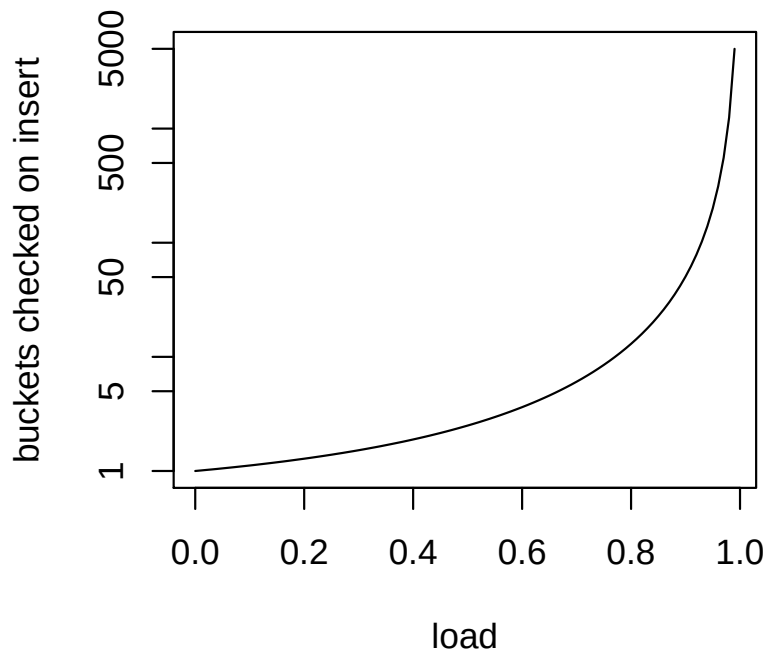
Theorem

The average number of cells examined during insertion with linear probing is

$$\frac{1}{2}\left(1 + \frac{1}{(1 - load)^2}\right)$$

Where,

$$load = \frac{\text{item count}}{\text{array length}}$$

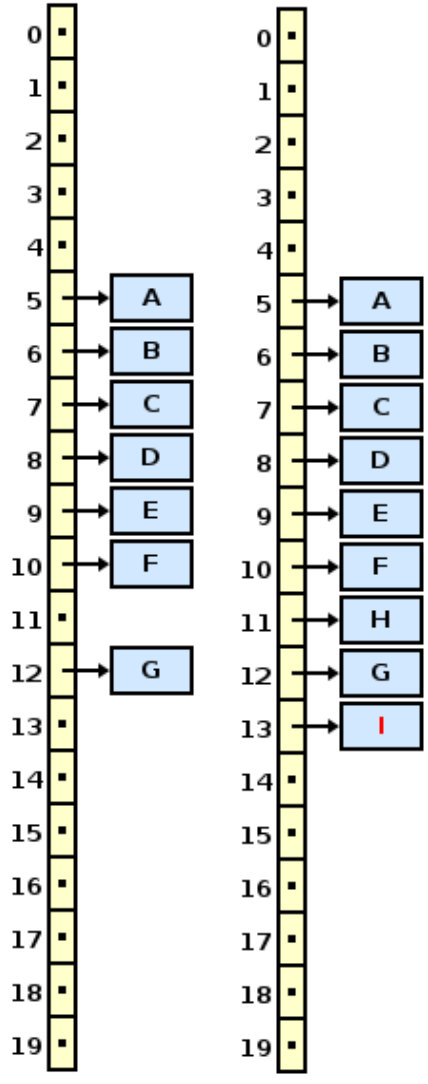


Why does this happen?

Primary Clustering

Many keys group together, clusters degrade performance

- Table size 20
- Filled cells 5-10, 12
- Insert H hashes to 6
  - Must put at 11
- Insert I hashes to 10
  - Must put at 13
- Hashes from 5-13 have clustered



Quadratic Probing

Try the following sequence until an empty array element is found

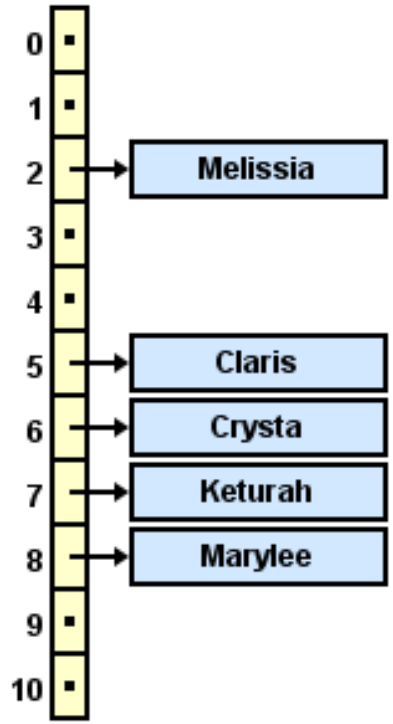
`pos, pos+1^2, pos+2^2, pos+3^2, ... pos+i^2`

- Primary clustering fixed: not putting in adjacent cells
- `add` works up to $load = 0.5$
  - Weiss Theorem 20.4, pg 786
- Can be done efficiently (Weiss pg 787)
- Complexity Not fully understood
  - No known relation of load to average cells searched
  - Interesting open research problem

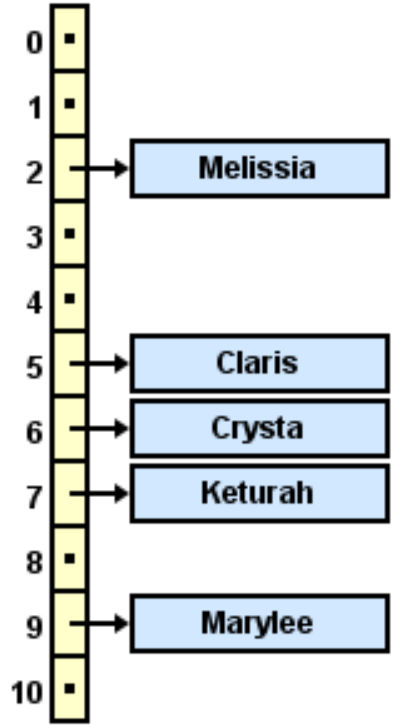
Probe Sequence Differences

```
> Math.abs("Marylee".hashCode()) % 11
5
```

Linear Probe



Quadratic Probe




```
> Math.abs("Barb".hashCode()) % 11
5 --> Where?
```

Rehashing

High load $\rightarrow$ make a bigger array, rehash, get small load

- Akin to expanding backing array in `ArrayList`
- Allocate a new larger array
- Copy over all active items to the new array
- Array should have prime number size
- $O(n)$ to rehash

Hash Tables in Java

`java.util.HashMap` `Map` built from hashing

`java.util.HashSet` `Set` built from hashing

`java.util.Hashtable` `Map` built from hashing, earlier class, *synchronized* for multithread apps

Hash Take-Home

- Provide $O(1)$ `add/remove/contains`
- Separate chaining is a pragmatic solution
  - Hash buckets have lists
- Open Address Hashing
  - Look in a sequence of buckets for an object
- Linear probing is one way to do open address hashing
  - Simple to implement: look in adjacent buckets
  - Performance suffers load approaches 1
  - Primary clustering hurts performance
- Quadratic probing is another way to do open address hashing
  - Prevents primary clustering
  - Must keep hash half-empty to guarantee successful `add`
  - Not fully understood mathematically

Hash Tables are *another* Container

Containers

- Like arrays, linked lists, trees, hash tables
- Have `add(x)`, `remove(x)`, `contains(x)` methods

`add(x)` put `x` in the DS
`removeLast()` get rid of "last" item
`remove(x)` take `x` out of DS
`contains(x)` is `x` in DS?

Speed Comparisons

- Speeds for array or `ArrayList`?
- Speeds for `LinkedList`?
- Speeds for hash table?

Operation Complexities (Speed)

- `add(x)`: put `x` in the DS
- `removeLast()`: get rid of "last" item
- `remove(x)`: take `x` out of DS
- `contains(x)`: is `x` in DS?

	add(x)	removeLast()	remove(x)	contains(x)
ArrayList	O(1)	O(1)	O(n)	O(n)
LinkedList	O(1)	O(1)	O(n)	O(n)
Hash Table	O(1)	X	O(1)	O(1)

This table is slightly misleading

- Careful of semantics of each operation
- Presence/lack of sorting property
- Set/Map distinctions
- What about space complexity of each?