CS 211: Recursion

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Week 13-1

Front Matter

Today

- P6 Questions
- Recursion, Stacks

Labs

- 13: Due today
- 14: Review and evals
- Incentive to attend lab 14, announce Tue/Wed

End Game

4/24	Mon	P6, Comparisons
4/26	Wed	Recursion
		Lab 13 Recursion
5/1	Mon	Stacks/Queues
		Lab 13 Due
5/3	Wed	Review/Evals
		Lab 14 Review/Evals
5/7	Sun	P6 Due
Mon	5/15	Final Exams
	002	10:30am-1:15pm
	006	1:30pm-4:15pm

Summarize Search Sort

- What are the built in search/sort routines in Java?
- What classes are they in?
- How can a new class be used with them?
- How fast are these library routines?
  - Linear search
  - Binary search
  - Sorting algorithm

Rabbits

A puzzle.[1]

> *Consider the growth of an idealized (biologically unrealistic) rabbit population, assuming that:*
>
> - *A newly born pair of rabbits, one male, one female, are put on an island;*
> - *Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits;*
> - *Rabbits never die and a mating pair always produces one new pair (one male, one female) every month from the second month on.*

How many pairs will there be in one year?

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[1] Adapted from Wikipedia

Simulation

Write a program to simulate the rabbit population.

- First we should develop a general approach
- Look at some data for this

Tabularly

Mature pair produce baby pair the following month

BN Baby pair from pair N

MN Mature pair from pair N

Month	0	1	2	3	4	5	6	7
Pairs	0	1	1	2	3	5	8	13
Pair 0		BI	MI	MI	MI	MI	MI	MI
Pair 1				B0	M0	M0	M0	M0
Pair 2					B0	M0	M0	M0
Pair 3						B0	M0	M0
Pair 4						B1	M1	M1
Pair 5							B0	M0
Pair 6							B1	M1
Pair 7							B2	M2
Pair 8								B0
Pair 9								B1
Pair 10								B2
Pair 11								B3
Pair 12								B4

Pattern

How does the population of a month relate to previous months?

Recursively

Population for Month *i* = Pop. Month *i-1* + Pop. Month *i-2*
Better known as *Fibonnaci Numbers*:

$$f_0 = 0$$

$$f_1 = 1$$

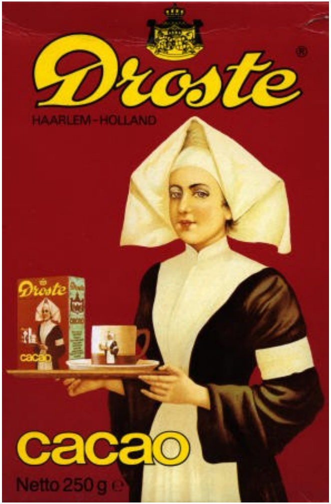
$$f_i = f_{i-1} + f_{i-2}$$

`public static int fib(int n)`

- Recursive implementation?
- Iterative implementation?
- Call Stack behavior in each

Recursion is. . .

Something specified in terms of a smaller version of itself



Recursion involves

Base Case

The "smallest thing", where you can definitively say "here is the answer"

Inductive/Recursive Case

If I had the answer to a few smaller versions of this problem, I could combine them to get the answer to this problem.

Identify Base and Recursive Cases

Fibonacci

$$f_0 = 0$$

$$f_1 = 1$$

$$f_i = f_{i-1} + f_{i-2}$$

Factorial

$$fact(n) = n * fact(n-1)$$

$$fact(0) = 1$$

Examine Stack Trace for Fibonacci

Recursive

```
public static int fibR(int n)
```

- Recursive implementation
- View Stack Trace of `fibR(4)`

Iterative

```
public static int fibI(int n)
```

- Iterative implementation?
- View Stack Trace of `fibI(4)`

Point

Recursion utilizes the Stack to store information about history

Exercise: Show the stack trace of fib

```
public class Fib{
  static int CALLS = 0;
  public static void main(String args[]){
    int fn = fib(4);
    System.out.printf("%d %d\n",fn,CALLS);
  }
  public static int fib(int n){
    CALLS++;
    // Draw call stack here when CALLS==9
    if(n==0){      return 0;    }
    if(n==1){      return 1;    }
    else{
      int tmp1 = fib(n-1);
      int tmp2 = fib(n-2);
      return tmp1+tmp2;
    }
  }
}
```

- static var `CALLS` counts number times `fib(n)` is entered
- Show stack trace starting with `fib(4)`
- Show local vars `n,tmp1,tmp2` in stack frames
- Stop when `CALLS` reaches 9

Other Uses for Recursion

Enumeration

Show me all possibilities of something

- All permutations of the numbers 1 to 10
- Print all games of Party Pong (hard problem from previous year)

Search Problems

Show me whether something exists and how its put together

- Does a number exist in an array?
- Does a path exist from point M to point C on a grid and what is that path?

```
||||||||||||||||||||||||
|M   ||    || ||  || C||
||| ||  | ||  |    | | |
|      ||        |     |
||||||||||||||||||||||||
```

Exercise: Sums

- Print all permutations of positive numbers which total 8 (order of numbers matters)
- Create a recursive helper called `totalsTarget()`
- Base and recursive cases?

Prototypes

```
public static void sumsTo8(){..}

public static
void totalsTarget(int target,
                  int current,
                  String history)
```

```
  target:  Eight!
  current: current total
  history: numbers used so far
```

Example output

```
> javac Sums.java
> java Sums
8 =  1 1 1 1 1 1 1 1
8 =  1 1 1 1 1 1 2
8 =  1 1 1 1 1 2 1
8 =  1 1 1 1 1 3
8 =  1 1 1 1 2 1 1
..
8 =  6 1 1
8 =  6 2
8 =  7 1
8 =  8
```

- 128 lines...
- Iterative version?

The "Power" of Recursion

Questions

- What problems can one solve with Recursion that *cannot* be solved with iteration (looping)
- Vice versa: loops can, recursion can't?

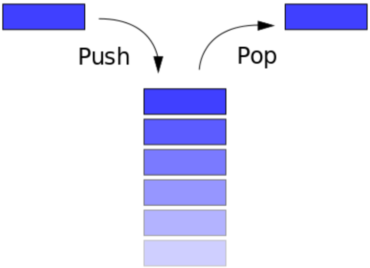
Stacks and Stacks of...

- We will shortly examine a solution to the sums problem which does not use recursion
- For that, we will need a data structure: a stack
- Should be familiar at this point based on our discussions of function call stack

Stacks

A data structure, supports a few operations

- `T s.getTop()`: return whatever is on top
- `s.push(T x)`: put `x` on top
- `void s.pop()`: remove whatever is on top
- `boolean s.isEmpty()`: `true` when nothing is in it, `false` o/w



Stacks are a LIFO:
Last In First Out

Questions

- Examples of stacks?
- How would you implement a stack using arrays?

Array Based Implementation of Stacks

- Must dynamically expand an internal array
- Following the textbook `ArrayList` implementation should make this easy
- Can check your work against `java.util.Stack`: should behave similarly

```
class AStack<T>{
  public AStack();                // Constructor
  public void push(T x);          // Like add(x)
  public T pop();                 // Like remove(size()-1)
  public T top();                 // Like get(size()-1)
  // peek() is often a synonym for top()
  public int size();
  public int getCapacity();
}
```

Sums to 8 - No Recursion

Consider again the sums-to-8 problem

```
> javac Sums.java
> java Sums
8 =  1 1 1 1 1 1 1 1
8 =  1 1 1 1 1 1 2
8 =  1 1 1 1 1 2 1
8 =  1 1 1 1 1 3
8 =  1 1 1 1 2 1 1
..
8 =  6 1 1
8 =  6 2
8 =  7 1
8 =  8
```

Use stacks to get the following

```
cur: 0 hist: '' toAdd: [8, 7, 6, 5, 4, 3, 2, 1]
cur: 1 hist: ' 1' toAdd: [7, 6, 5, 4, 3, 2, 1]
cur: 2 hist: ' 1 1' toAdd: [6, 5, 4, 3, 2, 1]
cur: 3 hist: ' 1 1 1' toAdd: [5, 4, 3, 2, 1]
cur: 4 hist: ' 1 1 1 1' toAdd: [4, 3, 2, 1]
cur: 5 hist: ' 1 1 1 1 1' toAdd: [3, 2, 1]
cur: 6 hist: ' 1 1 1 1 1 1' toAdd: [2, 1]
cur: 7 hist: ' 1 1 1 1 1 1 1' toAdd: [1]
cur: 8 hist: ' 1 1 1 1 1 1 1 1' toAdd: []
8 =  1 1 1 1 1 1 1 1
cur: 7 hist: ' 1 1 1 1 1 1 1' toAdd: []
cur: 6 hist: ' 1 1 1 1 1 1' toAdd: [2]
cur: 8 hist: ' 1 1 1 1 1 1 2' toAdd: []
8 =  1 1 1 1 1 1 2
...
...
8 =  6 2
cur: 6 hist: ' 6' toAdd: []
cur: 0 hist: '' toAdd: [8, 7]
cur: 7 hist: ' 7' toAdd: [1]
cur: 8 hist: ' 7 1' toAdd: []
8 =  7 1
```

Iterative Solutions

Use a little class to "simulate" a recursive call stack.

```
public static void totalsTarget(int target){
  Stack<SumFrame> stack = new Stack<SumFrame>();
  SumFrame first = new SumFrame(0,target,"");
  stack.push(first);

  // Simulate the recursive call stack with a loop
  while(stack.size() > 0){
    SumFrame frame = stack.peek();
```

Store info about what should be done at each step in those frames

```
class SumFrame{
  public int current;            // Current sum
  public Stack<Integer> toAdd;   // Numbers remaining to add
  public String history;         // History of adds that led here
```

Solution in `SumsNoRecursion.java`