

Homework #3

- (10pt) Prove that $\sum_{i=1}^L ix^i = \frac{x + (Lx - L - 1)x^{L+1}}{(1-x)^2}$
- (10pt) Find the $E[X^2]$ of the exponential distribution with parameter λ .
- (5pt) Using the result of the previous question to find the average queueing time W of a M/G/1 with exponentially distributed service times. (Hint: your result should be identical to the W of M/M/1.)
- (10pt) Verify that the solution of β in G/M/1 systems is $\sigma = \lambda/\mu$ when G is an exponential distribution with rate μ .

CS 756

1

- TCP uses the equation $RTT = RTT * w + New * (1-w)$ to compute the moving exponential average of round trip times, where New is a new sample of round trip time and w is a constant between 0 and 1. Suppose initially $RTT = R$ and all the subsequent samples are $2R$ (probably due to newly developed heavy traffic).
 - (8pt) Show that RTT_k , the value of RTT after receiving k samples, is $2R - R w^k$.
 - (7pt) Show that it takes $k = -1/\ln(w)$ packets for RTT_k to be $\geq (1-1/e)R + R$, that is, the gap between the real round trip time and the average round trip time to be less than around one third of R .

CS 756

2

- (10pt) Create an algorithm that generates a random variable having density function

$$f(x) = 20x(1-x)^3, \quad 0 < x < 1$$

Hint: Use the rejection method. An easy auxiliary random variable (the Y) is the uniform random variable from 0 to 1. Write $g(x)$, pdf of Y , and find an upper bound of $f(x)/g(x)$. You must show the steps to derive the upper bound c .