Physical Layer, Part 1
Communication Theory

Data Transmission

- A **signal** is an electrical or electromagnetic encoding of data
- **Signaling** is the act of propagating a signal along a medium
  - **guided** media: signals are sent along a physical path (e.g.,wire, cable, fiber)
  - **unguided** media: signals are broadcast (e.g., air, vacuum)
- A guided medium may be either
  - **point–to–point**: direct link between two devices
  - **multipoint**: more than two devices share the medium

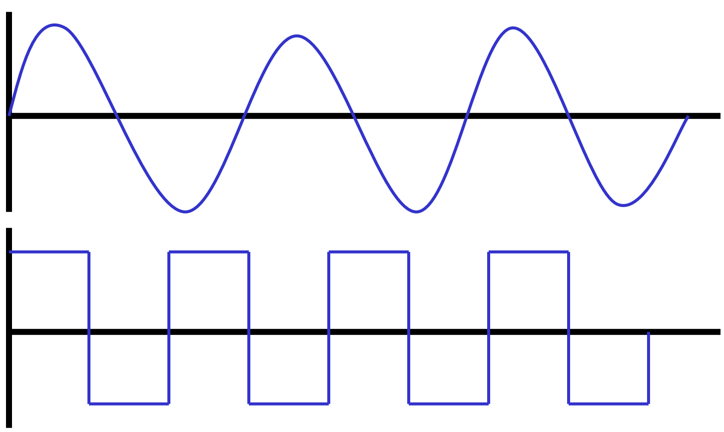
A Mathematical View of Signals

- A **signal** is a function of time.
- A signal $x(t)$ is **periodic** if and only if

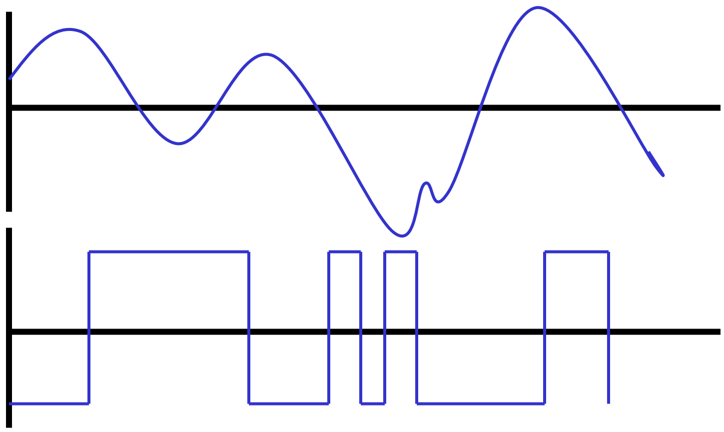
$$x(t+T) = x(t), \text{ for } -\infty < t < \infty$$

  Otherwise, it is **aperiodic.**
- Three characteristics of a periodic signal are
  - **amplitude**: the value of the signal at a time
  - **frequency**: inverse of the period T
  - **phase**: measure of the relative position in time within a signal period of a signal

Examples of Periodic Signals



Examples of Aperiodic Signals



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Fourier Analysis

- Any periodic signal can be represented as a sum of sinusoids, known as **Fourier series:**

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi n f_0 t) + \sum_{n=1}^{\infty} b_n \sin(2\pi n f_0 t)$$

where

$$a_0 = \frac{1}{T}\int_0^T x(t)dt$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cos(2\pi n f_0 t)dt$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin(2\pi n f_0 t)dt$$

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Fourier Analysis

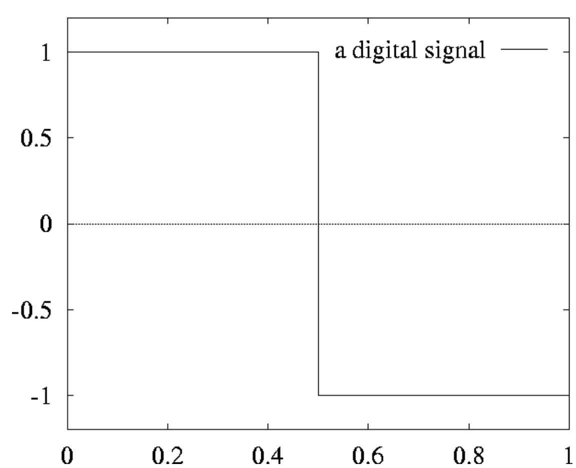
- f_0 is known as the **fundamental frequency**
- $T = 1/f_0$ is the **period** of the signal
- Multiples of f_0 are referred to as **harmonics**.
- The formula can be generalized to accommodate aperiodic signals

Speak"English," please:

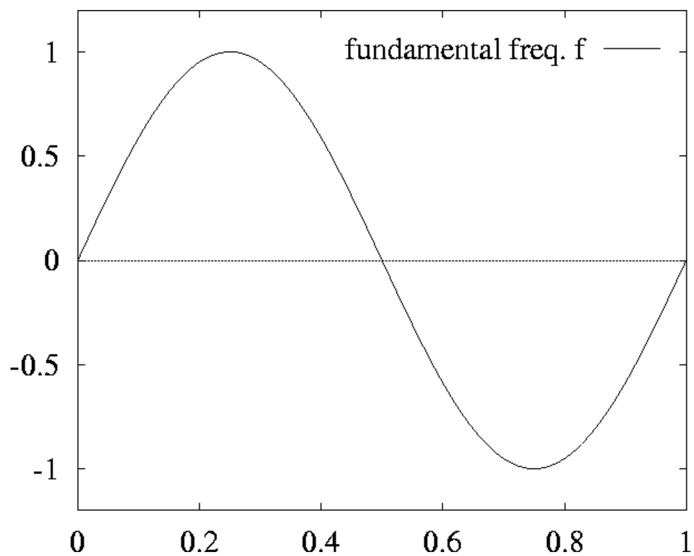
- We see any signal as the combined result of a (infinite) sequence of sinusoid functions, called **frequency components**.

Components of a Square Wave

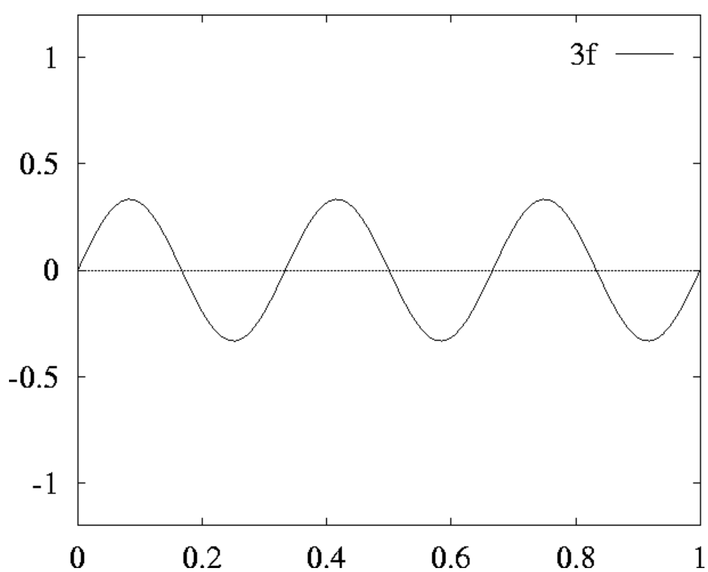
$$x(t) = \sin(2\pi \times ft) + \frac{1}{3}\sin(2\pi \times 3ft) + \frac{1}{5}\sin(2\pi \times 5ft) + \cdots$$



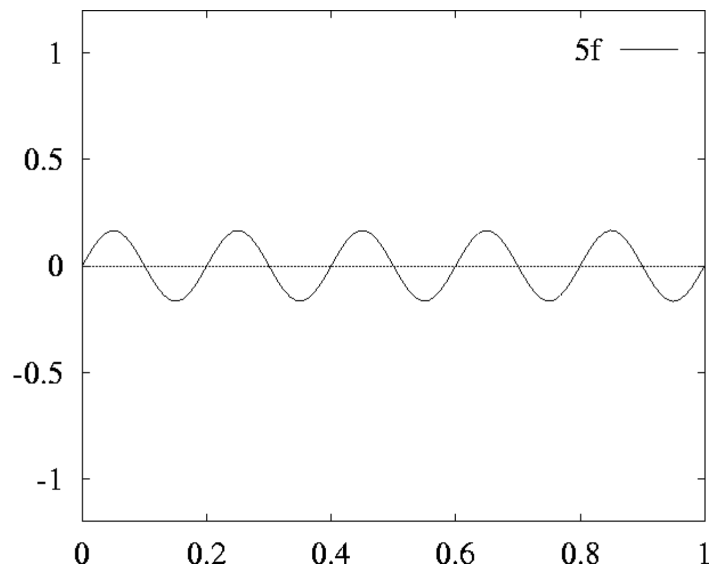
First Sin() Component



Third Component



Fifth Component



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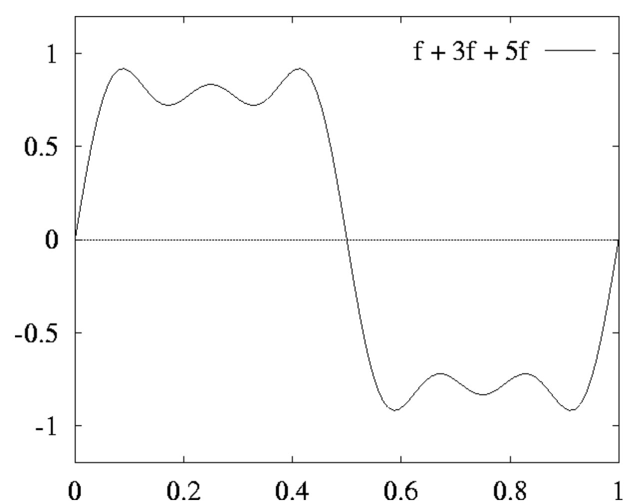
Combining Components



- The more high-frequency harmonics we include, the more faithful the result is to the original.

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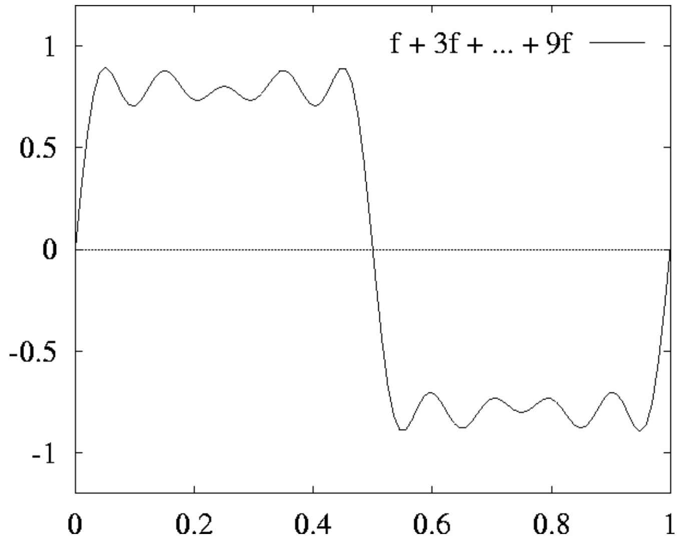
Even More Components



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Some Terminologies

- The **spectrum** of a signal is the range of frequencies that it contains.
- The absolute bandwidth is the width of the spectrum.
  - The absolute bandwidth of the square wave is infinite.
- Due to the limitations of real-world media, a signal must be represented in a limited band of frequencies. This band is referred to as the **effective bandwidth**, or just **bandwidth**.
- The exact range of this "limited band" is largely an engineering issue.

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Examples

- Consider a square wave $x(t)$ whose fundamental frequency f=1M Hz.
- If the representation of $x(t)$ by harmonics 1f+3f+5f is good enough, then the (effective) bandwidth of $x(t)$ is 5M – 1M = 4M Hz.
- A more faithful representation that uses up to 9f will have the bandwidth of 9M-1M = 8M Hz.

Bandwidth of Human Voice

- Typically, a baby can hear from 20 Hz to 20 KHz.
- Many adults, especially males, are not as capable.
  - Can you hear the 15 KHz noise produced by the CRT of your TV set ?
- Telephone systems pass frequencies from 300 Hz to 3300 Hz (bandwidth = 3000 Hz)
  - a transmission medium meeting this specification is called **voice grade**.

Nyquist Theorem

- Given a bandwidth *H*, the highest signal rate (the number of signaling elements per second) that can be carried is 2*H*.
- If each signal element contains *V* distinct values, than

$$\text{maximum data rate} = 2H \log_2 V \quad \text{bits/sec}$$

- This theorem assumes that the underlying medium is free of noises and, thus, gives an upper bound of the data rate.

Examples

- Consider a voice-grade line.
  - H = ?
- Using binary encoding, where each signal element could be either 0 or 1:
  - Max data rate = bits/sec
- Using a QAM encoding (studied later) that has 16 distinct values in each signaling element:
  - Max data rate = bits/sec

Transmission Impairment

- Attenuation
  - signal strength falls off with distance
  - attenuation increases with frequency
- Delay distortion
  - different frequency components propagate at different speeds over guided media
- Noise
  - cross talk: unwanted coupling between parallel signal paths
  - impulse noise: due to, for example, lighting

- Signal-to-Noise ratio is measured in **decibels**:

$$(S/N)_{dB} = 10 \times \log_{10} \frac{\text{signal power}}{\text{noise power}}$$

- Consequences
  - limited data rate or limited distance
  - errors in transmission inevitable

Shannon Theorem

$$\text{maximum data rate} = H \log_2(1 + S/N) \quad \text{bits/sec}$$

- Notice that we need the direct S/N ratio (not in decibel) in the formula.
- Example: H=3000Hz, S/N_{dB}=30
  - S/N = ?
  - Max data rate = ?
- Like Nyquist theorem, Shannon's theorem gives an upper bound.

More Examples

- Consider a voice grade communication link with S/N_{dB}=30 and V=4.
- According to Nyquist's theorem,
  - max data rate = ?
- According to Shannon's theorem,
  - max data rate = ?
- The maximum data rate of the link = ?
- If V=64, then the max data rate = ?
- The lesson: