$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{x}_i$$

2×2 covariance matrix:

$$E\left[(\boldsymbol{x}-\mu)(\boldsymbol{x}-\mu)^T\right] =$$

$$E\left[\begin{pmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{pmatrix}(x_1-\mu_1, x_2-\mu_2)\right] =$$

$$E\begin{bmatrix} (x_1-\mu_1)^2 & (x_1-\mu_1)(x_2-\mu_2) \\ (x_1-\mu_1)(x_2-\mu_2) & (x_2-\mu_2)^2 \end{bmatrix} =$$

$$\frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix} (x_1^i-\mu_1)^2 & (x_1^i-\mu_1)(x_2^i-\mu_2) \\ (x_1^i-\mu_1)(x_2^i-\mu_2) & (x_2^i-\mu_2)^2 \end{bmatrix}$$

$$\frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix}\left(x_1^i-\mu_1\right)^2 & \left(x_1^i-\mu_1\right)\left(x_2^i-\mu_2\right)\\ \left(x_1^i-\mu_1\right)\left(x_2^i-\mu_2\right) & \left(x_2^i-\mu_2\right)^2\end{bmatrix}=$$

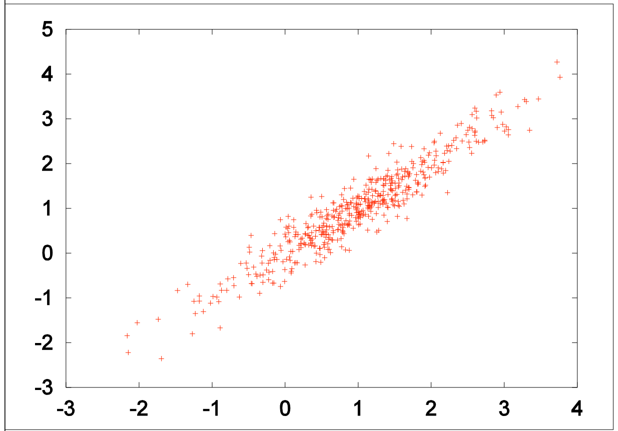
variance

covariance

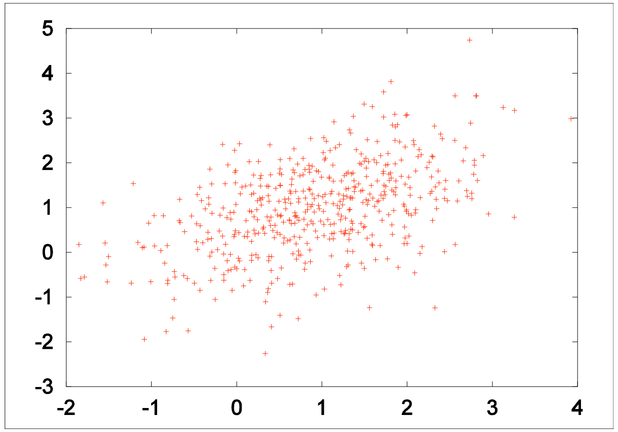
$$\begin{bmatrix}\frac{1}{N}\sum_{i=1}^{N}\left(x_1^i-\mu_1\right)^2 & \frac{1}{N}\sum_{i=1}^{N}\left[\left(x_1^i-\mu_1\right)\left(x_2^i-\mu_2\right)\right]\\ \frac{1}{N}\sum_{i=1}^{N}\left[\left(x_1^i-\mu_1\right)\left(x_2^i-\mu_2\right)\right] & \frac{1}{N}\sum_{i=1}^{N}\left(x_2^i-\mu_2\right)^2\end{bmatrix}$$

covariance

variance

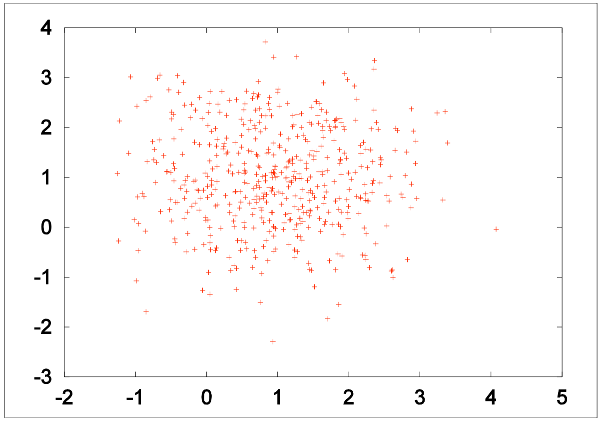


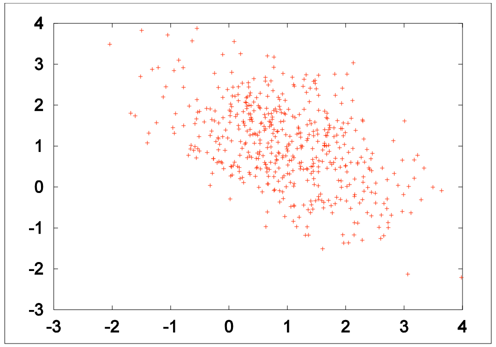
$$\begin{bmatrix} 0.94 & 0.93 \\ 0.93 & 1.03 \end{bmatrix}$$



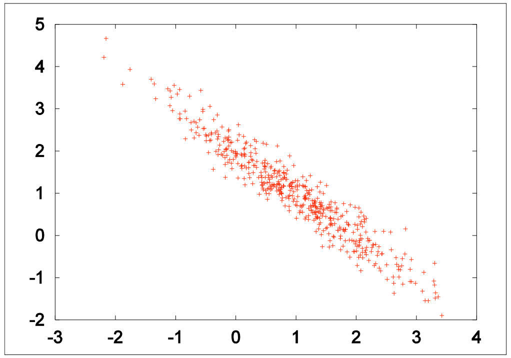
$$\begin{bmatrix} 0.97 & 0.49 \\ 0.49 & 1.04 \end{bmatrix}$$

$$\begin{bmatrix} 0.93 & 0.01 \\ 0.01 & 1.05 \end{bmatrix}$$





$$\begin{bmatrix} 0.99 & -0.5 \\ -0.5 & 1.06 \end{bmatrix}$$



$$\begin{bmatrix} 1.04 & -1.05 \\ -1.05 & 1.15 \end{bmatrix}$$