CS 330 (Fall 2013), Instructor: Carlotta Domeniconi

Quiz 1

Solutions

1. [30 points]

Determine whether the following statements are **T**rue or **F**alse.

$\{a, b\} \subseteq 2^{\{a,b,\{a,b\}\}}$ **FALSE**

$2^{\{a,b,\{a,b\}\}} = \{\emptyset, \{a\}, \{b\}, \{\{a, b\}\}, \{a, b\}, \{a, \{a, b\}\}, \{b, \{a, b\}\}, \{a, b, \{a, b\}\}\}$

So: $a \notin 2^{\{a,b,\{a,b\}\}}$ and $b \notin 2^{\{a,b,\{a,b\}\}}$. (Note that all elements of $2^{\{a,b,\{a,b\}\}}$ are sets.)
Therefore $\{a, b\}$ is not a subset of $2^{\{a,b,\{a,b\}\}}$.
What's TRUE is the following: $\{a, b\} \in 2^{\{a,b,\{a,b\}\}}$

$\{a, b, \{a, b\}\} - \{a, b\} = \{a, b\}$ **FALSE**

$\{a, b, \{a, b\}\} - \{a, b\} = \{\{a, b\}\}$ and $\{\{a, b\}\} \neq \{a, b\}$.
$\{\{a, b\}\}$ is a set with one element: $\{a, b\}$.
$\{a, b\}$ is a set with two elements: a and b.

$\emptyset \in \emptyset$ **FALSE**

By definition the *empty set* is the set that contains **NO** elements. It follows that the *empty set* cannot be an element of itself.

2. [30 points]

Define a binary relation P from $\Re$ to $\Re$ as follows:

$P = \{(x, y) | x \in \Re, y \in \Re, x = y^2\}$

Is P a function? Motivate your answer.

No, P is not a function. In fact:

$(4, 2) \in P$ since $4 = 2^2$
$(4, -2) \in P$ since $4 = (-2)^2$

So: the real number 4 is mapped into two different real numbers, i.e. 2 and -2. It follows that P is not a function.

3. [40 points]

Is $(p \wedge q) \vee r \equiv p \wedge (q \vee r)$ a valid equivalence? Use truth tables to motivate your answer.

No, the given equivalence is not valid. This is easily seen from the corresponding truth tables.