CS 330 (Fall 2013), Instructor: Carlotta Domeniconi

Quiz 3

Solutions:

1. [100 points]

Prove the following statement by mathematical induction:

$$\sum_{i=1}^{n+1} i \times 2^i = n \times 2^{n+2} + 2$$

for all integers $n \geq 0$.

Base case: clearly state what you need to prove as base case, and then prove it.

We need to show that the equality holds for $n = 0$:

When $n = 0$, $\sum_{i=1}^{n+1} i \times 2^i = \sum_{i=1}^{1} i \times 2^i = 2$ and $n \times 2^{n+2} + 2 = 2$.

Thus the equality holds for $n = 0$.

Inductive step: clearly write the inductive hypothesis and the inductive conclusion (i.e., the statement to be proved). Then proceed with the proof.

Inductive hypothesis:

$$\sum_{i=1}^{n+1} i \times 2^i = n \times 2^{n+2} + 2, n \geq 0$$

Inductive conclusion:

$$\sum_{i=1}^{n+2} i \times 2^i = (n+1) \times 2^{n+3} + 2$$

Proof:

$$\sum_{i=1}^{n+2} i \times 2^i = \sum_{i=1}^{n+1} i \times 2^i + (n+2) \times 2^{n+2} = n \times 2^{n+2} + 2 + (n+2) \times 2^{n+2}$$

by the inductive hypothesis

$$= (n + n + 2) \times 2^{n+2} + 2 = (2n + 2) \times 2^{n+2} + 2 = (n+1)2^{n+3} + 2$$

This proves the inductive conclusion.