CS 330 (Fall 2013), Instructor: Carlotta Domeniconi

Quiz 2

**Solutions**

1. [50 points]

Prove the following equivalence by substitution, i.e., use known logical equivalences to show that $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q)$ is equivalent to $\neg p$. *You must start from the statement* $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q)$. Justify each step with the name of the corresponding logical equivalence being used. For full credit, do *not* skip steps.

$$\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv \neg p$$

| | | |
|---|---|---|
| $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q)$ | $\equiv (\neg p \wedge q) \vee (\neg p \wedge \neg q)$ | De Morgan's law |
| | $\equiv \neg p \wedge (q \vee \neg q)$ | distributive law |
| | $\equiv \neg p \wedge \text{TRUE}$ | excluded middle law |
| | $\equiv \neg p$ | property of the constant TRUE |

2. [50 points]

Using rules of inference with no substitution, prove that *modus ponens* is a valid rule. Use the notation introduced in class, and state, for each line, the rule of inference that justifies it. Make sure you do *not* to use *modus ponens* itself in your proof!

*Modus ponens*:

$p \rightarrow q$

$p$

———

$q$

**Write below the equivalent expression you need to prove**:

**Equivalent expression**:

$((p \rightarrow q) \wedge p) \rightarrow q$

Proof:

| | |
|---|---|
| $[(p \to q) \land p]$ | Assumption |
| $p \to q$ | $\land$ elimination |
| $p$ | $\land$ elimination |
| $\quad [\neg q]$ | Assumption |
| $\quad \neg p$ | Modus tollens |
| $\quad$ FALSE | Contradiction |
| $q$ | Reduction to absurdity |
| $((p \to q) \land p) \to q$ | $\to$ introduction |