CS 330 Formal Methods and Models

Quiz 4 (Fall 2010)

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Solutions

This test is governed by the GMU Honor Code. The paper you turn in must be your sole work. Help may be obtained from the instructor to understand the description of the problem, but the solution must be the student's own work. Any deviation from this is considered a Honor Code violation.

1. [50 points]

Use DeMorgan's laws to write the negation of each of the following statements:

$\forall x \in \mathbb{R} : (x > \frac{1}{x})$

$\neg(\forall x \in \mathbb{R} : (x > \frac{1}{x})) \equiv \exists x \in \mathbb{R} : (x \leq \frac{1}{x})$

$\exists x \in \mathbb{R} : (x^2 = 2)$

$\neg(\exists x \in \mathbb{R} : (x^2 = 2)) \equiv \forall x \in \mathbb{R} : (x^2 \neq 2)$

$\forall x \in \mathbb{R} : ((x > 3) \rightarrow (x^2 > 9))$

$\neg(\forall x \in \mathbb{R} : ((x > 3) \rightarrow (x^2 > 9))) \equiv \exists x \in \mathbb{R} : ((x > 3) \wedge (x^2 \leq 9))$

2. [50 points]

Consider the statement given below. Write a new statement by interchanging the quantifiers $\forall$ and $\exists$. State which is true: the given statement, the version with interchanged quantifiers, neither, or both. Justify your answer.

$\forall x \in \mathbb{R} : \exists y \in \mathbb{R} : (x < y)$

The given statement is true. It says that, for any given real number x, there exist a real number y that is greater than x. This is indeed true because, given an arbitrary real x, we can choose $y = x + 1$. y is a real number, and $y > x$.

Version with interchanged quantifiers:

$\exists y \in \mathbb{R} : \forall x \in \mathbb{R} : (x < y)$

This statement is false. It says that there exist a "special" real number y with the property of being greater than all real numbers. This is clearly false: regardless of how we pick y, it always exists a real number greater than y (e.g., $x = y + 1$).