CS 688 – Spring 2016

Homework 3 – Due March 28

Professor: Carlotta Domeniconi

Problem 1 Consider the following kernel function: $K(\mathbf{x}_i, \mathbf{x}_j) = (< \mathbf{x}_i, \mathbf{x}_j >)^2$. Verify that for each of the following two mappings ϕ, it holds $K(\mathbf{x}_i, \mathbf{x}_j) =< \phi(\mathbf{x}_i), \phi(\mathbf{x}_j) >$. Show your calculations.

1. $\phi : \Re^2 \rightarrow \Re^3$, $\phi(\mathbf{x}) = \frac{1}{\sqrt{2}} \begin{pmatrix} x_1^2 - x_2^2 \\ 2x_1x_2 \\ x_1^2 + x_2^2 \end{pmatrix}$

2. $\phi : \Re^2 \rightarrow \Re^4$, $\phi(\mathbf{x}) = \begin{pmatrix} x_1^2 \\ x_1x_2 \\ x_1x_2 \\ x_2^2 \end{pmatrix}$

Problem 2 Consider the kernel function:

$$K(\mathbf{x}, \mathbf{y}) = \mathbf{x} \cdot \mathbf{y} + 4(\mathbf{x} \cdot \mathbf{y})^2$$

where the vectors $\mathbf{x}$ and $\mathbf{y}$ are two-dimensional vectors. This kernel is equal to an inner product $\phi(\mathbf{x}) \cdot \phi(\mathbf{y})$ for some definition of ϕ. What is the function ϕ?

Problem 3 Prove that the parity function of $n > 2$ binary inputs $x_1, x_2, \ldots, x_n$ cannot be computed by a perceptron. A parity function is a Boolean function whose value is 1 if and only if the input vector has an odd number of ones.

Problem 4 Consider the logistic regression classifier and the following quantity, called the *log-odds of success*:

$$\ln \frac{P(Y = 1|x)}{P(Y = 0|x)}$$

Show that the log-odds of success is a linear function of x.

Instructions This homework is due on March 28, before the beginning of class. **Turn in a hardcopy before the beginning of class**.