

# CS 689 – Robot Motion Planning

## Manipulation Planning

Amarda Shehu

Department of Computer Science  
George Mason University

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[movie: industrial]

[movie: L-shape]

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How is manipulation planning a motion planning problem?

- What moves where?
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# What is Manipulation Planning?

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How is manipulation planning a motion planning problem?

- What moves where?
- Workspace?
- Configuration space?
  - Need to keep track of ? and ? moving in workspace?

# Problem Formulation

Given:

- a description of the obstacles
- a description of the robot manipulator
- a description of the object to be manipulated
- a description of the initial and desired placements for the object

Objective:

- compute a sequence of motions where the robot manipulator grasps the object in its *initial placement* and places it in its *desired placement* while *avoiding collisions*

## Some Challenges

- How to grasp the object?
- Is the grasp stable?
- Does the solution require re-grasping?
- When should the robot manipulator release the object and re-grasp it in a different configuration?



# Two Representative Approaches

PRM-based: Nielsen and Kavraki, IROS 2000.

- Expands roadmap/graph to manipulation graph.
- Assumes stable robot grasps and object placements pre-computed and provided ahead of time.

RRT-based: Berenson et al., ICRA 2009.

- Approaches it as an inverse kinematics problem.
- Enriches any provided object placements with more and computes new robot grasps.

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Focus: efficient construction of manipulation graph.

- Observation on whether motion of robot is with object grasped or not.

# Observations

- Solution path consists of a sequence of transfer and transit paths
- Transfer path: subpath where object is stably grasped and moved by robot
- Transit path: subpath where object is left in a stable position while robot changes grasp

# Manipulation Graph: Vertices

Each node is a triple  $(q_{\text{obj}}, g, q_{\text{rob}})$ , where:

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- $q_{\text{obj}}$  specifies a stable placement (position and orientation) of the object
  - Provided or pre-computed before construction of graph
- $g$  specifies a position and orientation of the robot tool relative to the placement of the object at which the tool is able to grasp the object
  - Provided before construction of graph
- $q_{\text{rob}}$  is the configuration of the robot for which the robot tool is able to grasp the object placed at  $q_{\text{obj}}$  using the grasp  $g$ 
  - Focus of this approach



# Manipulation Graph: Edges

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An edge  $((q_{\text{obj}}^i, g, q_{\text{rob}}^i), (q_{\text{obj}}^j, g, q_{\text{rob}}^j))$  indicates a **transfer** (local) path where the object is grasped according to  $g$  and the robot moves with the object from configuration  $(q_{\text{obj}}^i, q_{\text{rob}}^i)$  to  $(q_{\text{obj}}^j, q_{\text{rob}}^j)$

## Manipulation Graph: Edges

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An edge  $\left( (q_{\text{obj}}, g^i, q_{\text{rob}}^i), (q_{\text{obj}}, g^j, q_{\text{rob}}^j) \right)$  indicates a **transit** (local) path where the object is left at a stable placement  $q_{\text{obj}}$  while the robot changes grasp from  $(g^i, q_{\text{rob}}^i)$  to  $(g^j, q_{\text{rob}}^j)$

## PRM Approach

- Node Generation:

**for**  $i = 1, \dots, N$  **do** sample a node  $(q_{\text{obj}}^i, \mathbf{g}^i, q_{\text{rob}}^i)$

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connect neighboring nodes  $\left( (q_{\text{obj}}^i, g^i, q_{\text{rob}}^i), (q_{\text{obj}}^j, g^j, q_{\text{rob}}^j) \right)$

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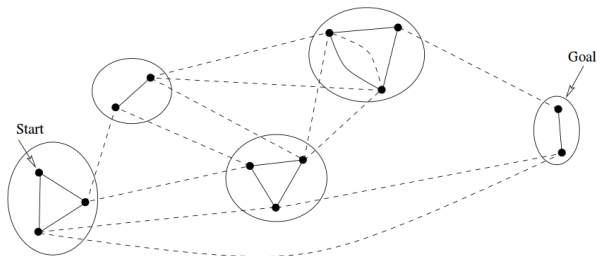
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- Edge Generation:

connect neighboring nodes  $((q_{\text{obj}}^i, g^i, q_{\text{rob}}^i), (q_{\text{obj}}^j, g^j, q_{\text{rob}}^j))$

How is local path generated for transfer or transit edge?

# Manipulation Graph



Solid lines represent transit paths, and dotted lines represent transfer paths.

## Challenges:

- Each edge generation gives rise to a path-planning problem
- Must verify edge validity before adding it to manipulation graph
- Too many edge verifications (since graph could have large number of nodes)

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## FuzzyPRM Idea

- Probabilistic edges instead of deterministic edges
- Use a probabilistic path planner to compute edge connections
- Probability associated with an edge  $e$  depends on the time spent by probabilistic path planner on  $e$
- From the people that gave you the Lazy PRM...

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# A two-level Fuzzy PRM for Manipulation Planning

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## Low-Level Fuzzy PRM

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- 7: **repeat**
- 8:  $(q', q'') \leftarrow$  edge in  $\phi$  with lowest  
probability
- 9: **if**  $\text{prob}(q', q'') \neq 1$  **then**
- 10: run subdivision collision checking to  
validate  $(q', q'')$  at resolution  
 $\ell(q', q'')$
- 11: increment  $\ell(q', q'')$

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## Query Stage

- 1: **while** no solution found **do**
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- 5:       run low-level fuzzy PRM on  $e$  for a short period of time
- 6:       **if** success **then**
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## Low-Level Fuzzy PRM

- 1: **if** mode = "CONSTRUCTION" **then**
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- 4:    $\text{prob}(q, q') \leftarrow P^*(l)$
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- 7:   **repeat**
- 8:      $(q', q'') \leftarrow$  edge in  $\phi$  with lowest probability
- 9:     **if**  $\text{prob}(q', q'') \neq 1$  **then**
- 10:       run subdivision collision checking to validate  $(q', q'')$  at resolution  $\ell(q', q'')$
- 11:       increment  $\ell(q', q'')$
- 12:       **if** collision **then**
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## Manipulation Graph

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- 16:     **until** all edges in  $\phi$  have prob 1
- 17:     return success



# Manipulation Planning with Workspace Goal Regions

[Berenson, Srinivasa, Ferguson, Collet, Kuffner: ICRA 2009]

- Manipulation planners often require specification of a set of stable grasp configurations

# Manipulation Planning with Workspace Goal Regions

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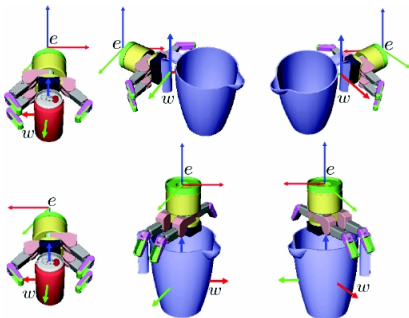
- Manipulation planners often require specification of a set of stable grasp configurations
- This forces the planner to use only these configurations as goals
- If the chosen goal configurations are unreachable, the planner will fail
- Even when reachable, it may take the planner a long time to find solutions to these goal configurations

# Manipulation Planning with Workspace Goal Regions

[Berenson, Srinivasa, Ferguson, Collet, Kuffner: ICRA 2009]

## Proposed Approach

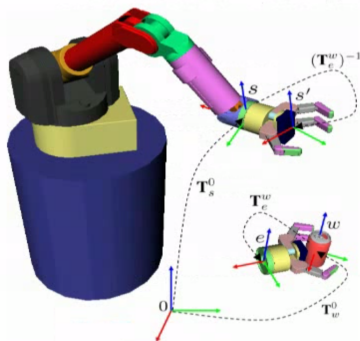
- Introduce concept of Workspace Goal Regions (WGRs)
- WGR allows the specification of continuous regions in the six-dimensional workspace of end-effector poses as goals for the planner



- Two WGRs describe grasping a soda can
- Bounds allow rotation around z axis of w

# Definition of a WGR

- Reference frame  $w$  attached at object specifying pre-computed *grasp pose*
- Workspace bounds  $B^w$  specifying flexibility around target grasp  $w$ :  
[[ $x_{\min}, x_{\max}$ ], [ $y_{\min}, y_{\max}$ ], [ $z_{\min}, z_{\max}$ ],  
[ $\psi_{\min}, \psi_{\max}$ ], [ $\theta_{\min}, \theta_{\max}$ ], [ $\phi_{\min}, \phi_{\max}$ ]]



- To allow offset for end-effector, transform  $T_e^w$  specifies end-effector pose relative to the ( $w$ ) reference frame of the desired grasp
- Simple operations can be done:  $T_w^0 T_e^w$  now specifies a target pose of end effector in world coordinate frame
- One can sample alternative pose for end effector from  $B^w$ , and then convert to world coordinate frame to provide an end-effector goal pose to IK solver

## Why Use WGRs for Manipulation Planning

- Sampling from  $B^w$  (in the provided range for each of the 6 coordinates that specify the pose of target, pre-specified grasp) gives alternative grasper pose in (w/object's) coordinate frame.
- Sample can be converted into new, sampled goal pose for end-effector.
- IK can be used to steer manipulator towards sampled goal end-effector pose.
- All encapsulated in an IK bi-directional RRT (IKBiRRT) so as to deal with the usual get-stuck (suboptimal) behavior of gradient-descent type methods for IK.
- A distance measure can be specified to give a sense of how far or near two end-effector configurations are for RRT.

# Some Sampling Target End-Effector Pose from WGR

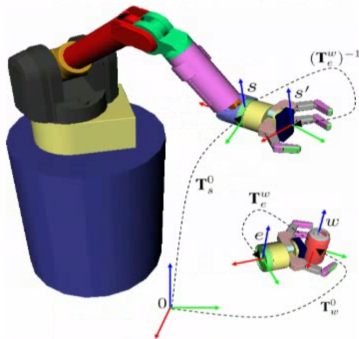
- $d_{\text{sample}}^w \leftarrow \text{sample}$  a random value between each of the bounds defined by  $B^w$  with uniform probability
- convert  $d_{\text{sample}}^w$  into a transformation matrix  $T_{\text{sample}}^w$ , which specifies the *sampled grasper pose* relative to the coordinate frame  $w$  of the target grasp.
- convert the *sampled grasper pose* into a *sampled pose for the end-effector*, still in the coordinate frame of  $w$  (target grasp pose)

$$T_{\text{sample}}^w \cdot T_e^w$$

- convert the sampled end-effector pose in world coordinates

$$T_{\text{sample}'}^0 = T_w^0 T_{\text{sample}}^w T_e^w$$

- $T_{\text{sample}'}^0$  is passed to an IK solver to generate solution(a)s, which are checked for collisions. Only collision-free solutions are added to the RRT.



# Distance Measurement for RRT

- use FK to get end-effector pose at current  $q_s$  configuration:  $T_s^0$  is pose of end-effector in world coordinates.
- get pose of grasp, if object held there, in world coordinates

$$T_{s'}^0 = T_s^0 (T_e^w)^{-1}$$

- convert it from world to coordinates of  $w$

$$T_{s'}^w = (T_w^0)^{-1} T_{s'}^0$$

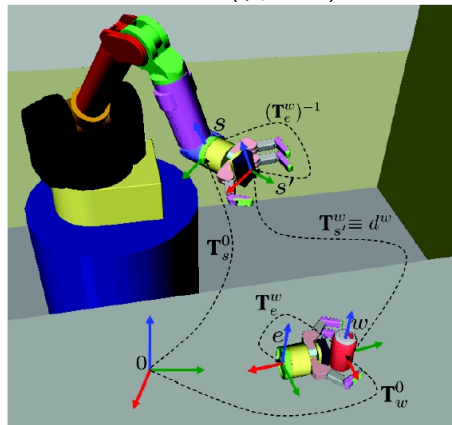
- convert  $T_{s'}^w$  into a  $6 \times 1$  displacement vector from origin of  $w$  frame

$$d^w = \begin{bmatrix} t_{s'}^w \\ \arctan2(R_{s'32}^w, R_{s'33}^w) \\ -\arcsin(R_{s'31}^w) \\ \arctan2(R_{s'21}^w, R_{s'11}^w) \end{bmatrix}$$

- take into account bounds  $B^w$  to get  $6 \times 1$  displacement vector  $\Delta x$  from  $d^w$

$$\Delta x_i = \begin{cases} d_i^w - B_{i,1}^w & \text{if } d_i^w < B_{i,1}^w \\ d_i^w - B_{i,2}^w & \text{if } d_i^w > B_{i,2}^w \\ 0 & \text{otherwise} \end{cases}$$

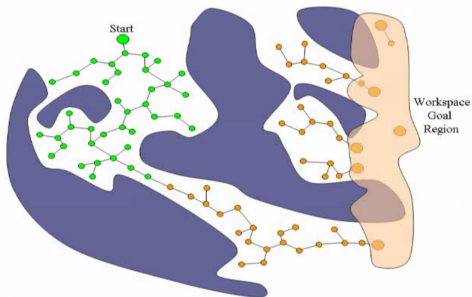
Distance to WGRs:  $d(q_s, WGR)$



$$d(q_s, WGR) = \|\Delta x\|$$

# Inverse Kinematics Bi-Directional RRT (IKBiRRT): Overall Approach

- Grows one tree from start and one tree from goal configuration.
- At each iteration chooses between one of two modes: exploration through standard BiRRT and sampling from the set of WGRs  $W$ . The probability of choosing the mode is controlled by the parameter  $P_{\text{sample}}$ .
- Goal configurations sampled from a WGR are injected into the backwards tree that grows from goal.
- Termination when both trees meet at some configuration.





# Inverse Kinematics Bi-Directional RRT (IKBiRRT)

```
1:  $T_a$ .INIT( $q_s$ );  $T_b$ .INIT(NULL)
2: while TIMEREMAINING() do
3:    $T_{\text{goal}} \leftarrow \text{GETBACKWARDTREE}(T_a, T_b)$ 
4:   if  $T_{\text{goal}}$ .size() = 0 or  $\text{rand}(0, 1) < P_{\text{sample}}$  then
5:     ADDIKSOLUTIONS( $T_{\text{goal}}$ )
6:   else
7:      $q_{\text{rand}} \leftarrow \text{RANDCONFIG}()$ 
8:      $q_{\text{near}}^a \leftarrow \text{NEARESTNEIGHBOR}(T_a, q_{\text{rand}})$ 
9:      $q_{\text{reached}}^a \leftarrow \text{EXTENDTREE}(T_a, q_{\text{near}}^a, q_{\text{rand}})$ 
10:     $q_{\text{near}}^b \leftarrow \text{NEARESTNEIGHBOR}(T_b, q_{\text{rand}})$ 
11:     $q_{\text{reached}}^b \leftarrow \text{EXTENDTREE}(T_b, q_{\text{near}}^b, q_{\text{rand}})$ 
12:    if  $q_{\text{reached}}^a = q_{\text{reached}}^b$  then
13:      return EXTRACTPATH( $T_a, q_{\text{reached}}^a, T_b, q_{\text{reached}}^b$ )
14:    else
15:      SWAP( $T_a, T_b$ )
```

[movie]