Lecture 2: Problem Solving and (Uninformed) Search

CS 580 (001) - Spring 2018

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1. Outline of Today's Class

2. Problem-solving Agents

3. Problem Types

4. Problem Formulation

5. Example Problems

6. Elementary (Graph) Search Algorithms
   - Uninformed Search
     - Breadth-first Search (BFS)
     - Depth-first Search (DFS)
     - Depth-limited Search (DLS)
     - Iterative Deepening Search (IDS)

Restricted Form of a General Agent

```
function SIMPLE-PROBLEM-SOLVING-AGENT( percept) returns an action
    static: seq, an action sequence, initially empty
            state, some description of the current world state
            goal, a goal, initially null
            problem, a problem formulation

    state ← UPDATE-STATE(state, percept)
    if seq is empty then
        goal ← FORMULATE-GOAL(state)
        problem ← FORMULATE-PROBLEM(state, goal)
        seq ← SEARCH( problem)
    action ← RECOMMENDATION(seq, state)
    seq ← REMAINDER(seq, state)
    return action
```

Note: this is **offline** problem solving; solution executed “eyes closed.”
Online problem solving involves acting without complete knowledge.

Example: Romania

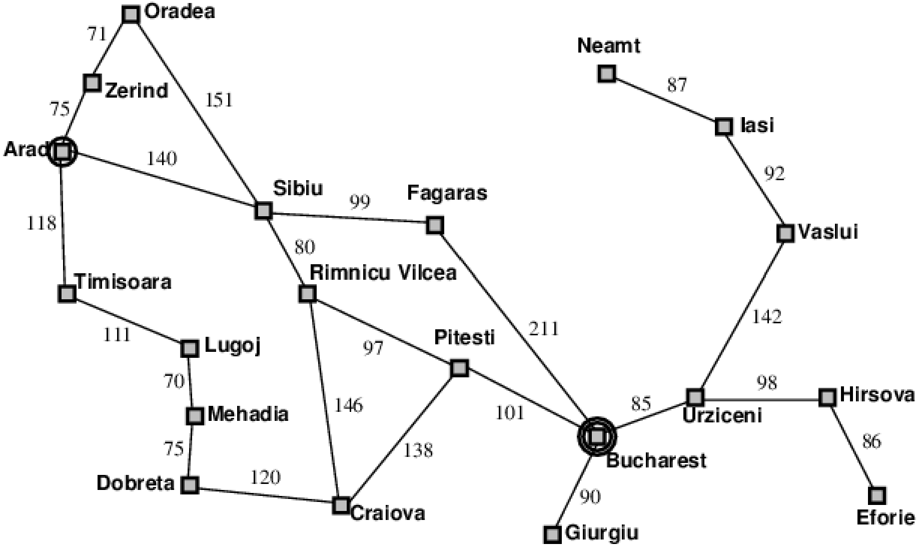
On holiday in Romania; currently in Arad.
Flight leaves tomorrow from Bucharest.

Formulate goal:
be in Bucharest

Formulate problem:
states: various cities
actions: drive between cities

Find solution:
sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania



Problem Types

- **Fully-observable, Known, Deterministic** → single-state problem

  Agent knows exactly which state it will be in; solution is a sequence of actions that can be executed eyes closed
  open loop: no need to sense environment during execution

- **Non-observable** → conformant problem

  Agent may have no idea where it is; solution (if any) is a sequence
  Also known as multi-state problem: agent knows which states it might be in

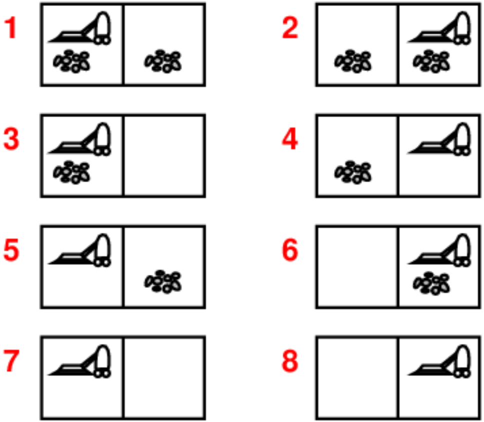
- **Nondeterministic** and/or **Partially observable** → contingency problem

  Percepts provide **new** information about current state
  Solution is a contingent plan or a policy
  Often **interleave** search, execution
  plans contain conditional parts based on sensors

- **Unknown environment** → exploration problem ("online")

  Agent must learn the effect of its actions

Example: Vacuum World



Single-state, start in #5.

Solution??

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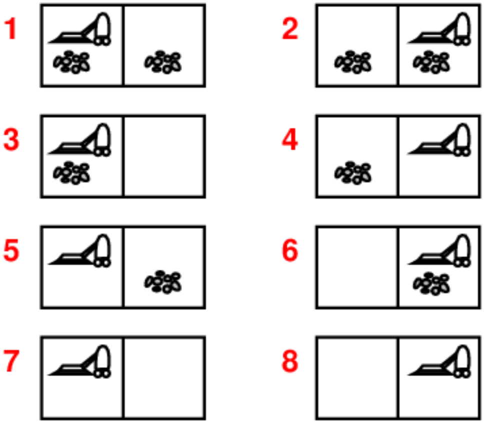

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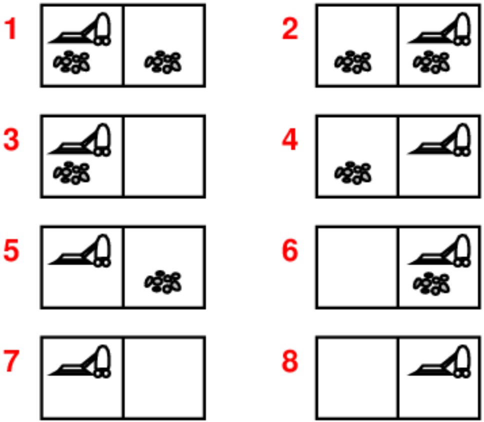

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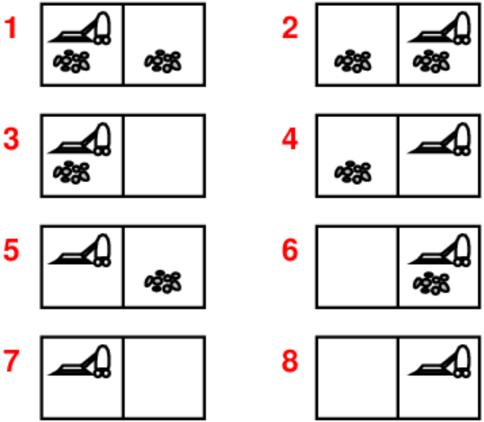

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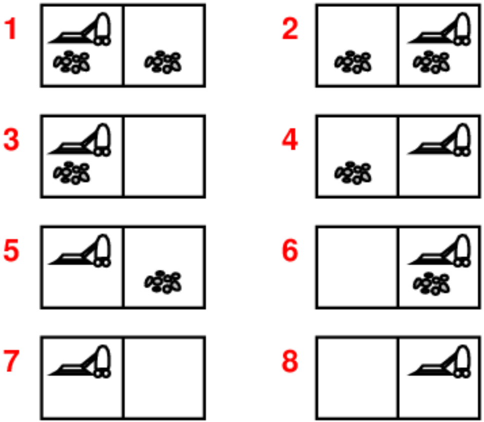

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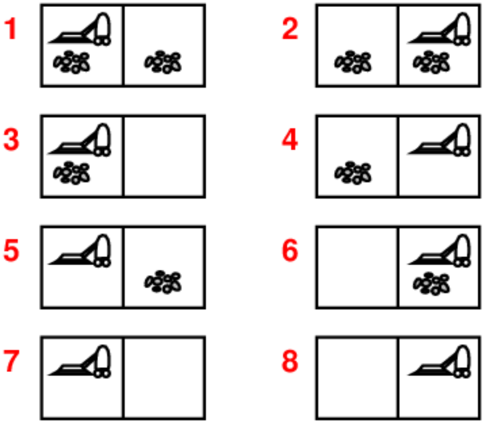

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Murphy's Law: *Suck* can dirty a clean carpet
Local sensing: dirt, location only.

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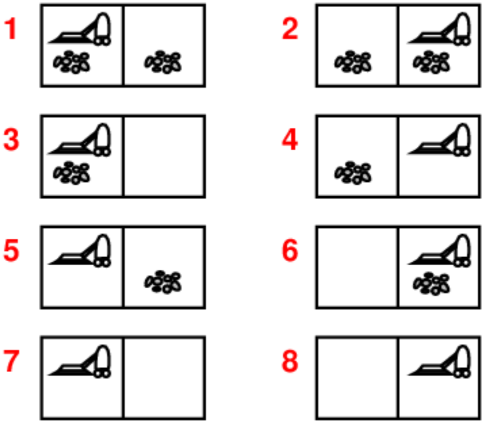

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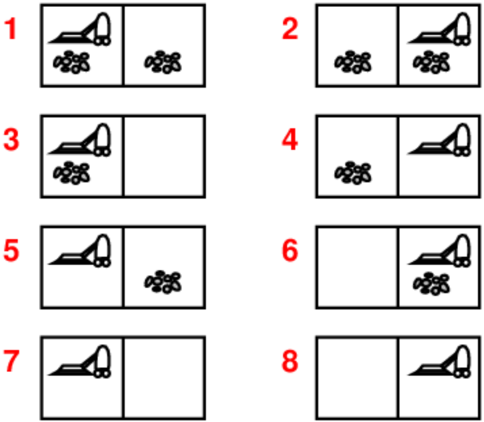

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Single-state Problem Formulation for Route-Finding

A problem is defined by five components:

1. **Initial state** e.g., “In(Arad)”
2. **Actions** e.g.
   ACTION(Arad) = { Arad → Timisoara, Arad → Sibiu, . . ., Arad → Zerind }
3. **Transition model**
   e.g. RESULT(Arad, Arad → Zerind) = Zerind
4. **Goal test**, can be:
   - **explicit** e.g., “In(Bucharest)”
   - **implicit** e.g., $NoDirt(s)$
5. **Path cost** (additive)
   e.g. sum of distances, number of actions executed, etc.
   - $c(x, a, y)$ is the step cost, assumed to be ≥ 0

Solution:

A **solution** is a sequence of actions leading from the initial state to a goal state
the process of looking for a solution is called **search**

Abstraction: Selecting a State Space

Real world is absurdly complex

⇒ state space must be **abstracted** for problem solving
(Abstract) state = set of real states
(Abstract) action = complex combination of real actions

e.g., "Arad → Zerind" represents a complex set
of possible routes, detours, rest stops, etc.

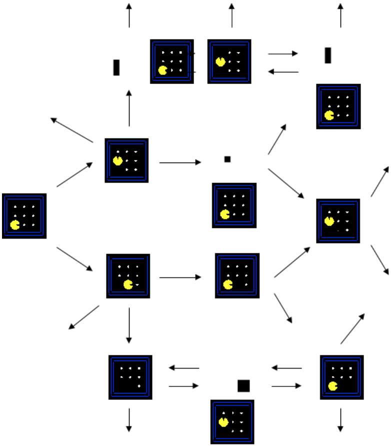
For guaranteed realizability, **any** real state "in Arad"
must get to **some** real state "in Zerind"

(Abstract) solution =
set of real paths that are solutions in the real world

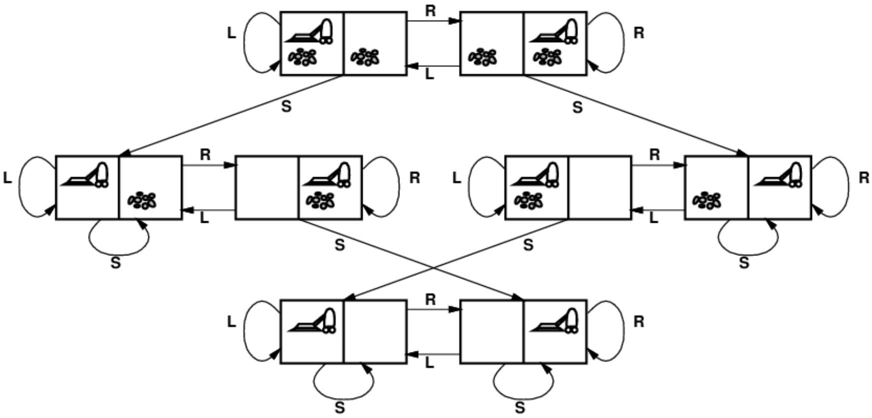
Each abstract action should be "easier" than the original problem!

State Space Graph

- State space graph: A mathematical representation of a search problem
- Nodes are (abstracted) world configurations
- Arcs/edges/links represent successors (action results)
- Goal test is a set of goal nodes (maybe only one)
- In a state space graph, each state occurs only once!
- We can rarely build this full graph in memory (its too big), but it's a useful idea



Example: Vacuum World State Space Graph



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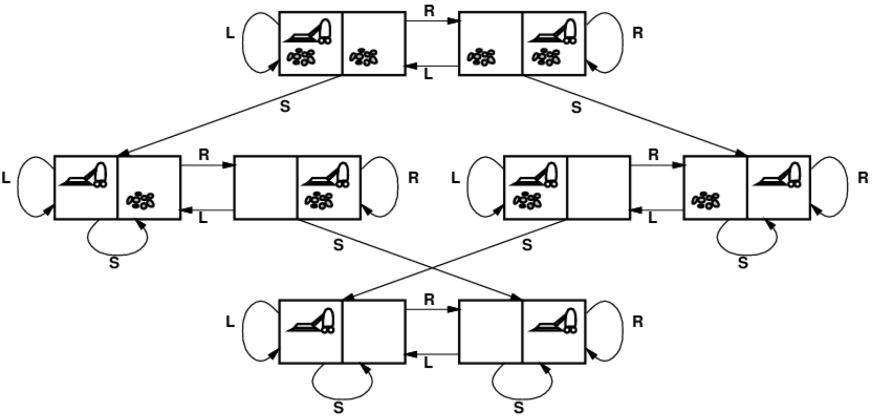

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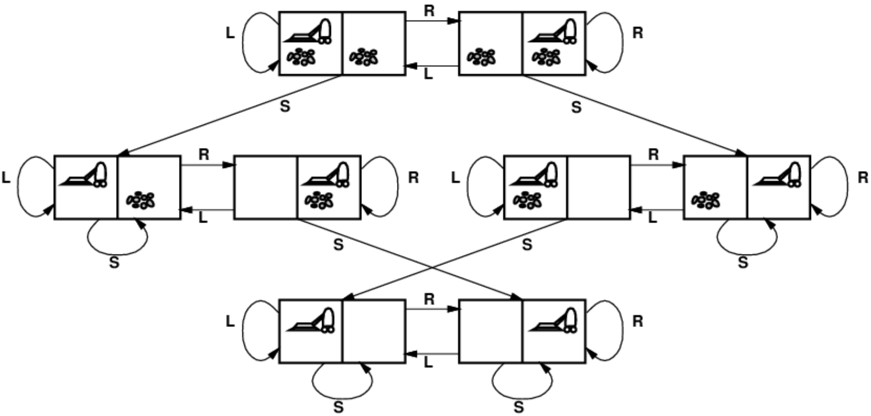

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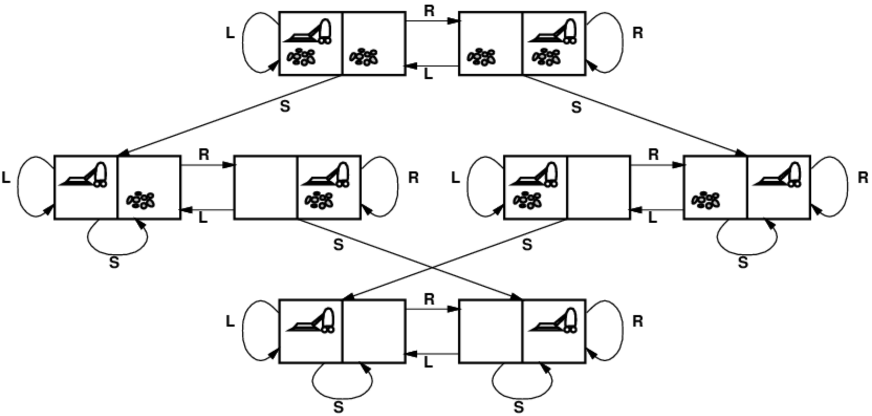

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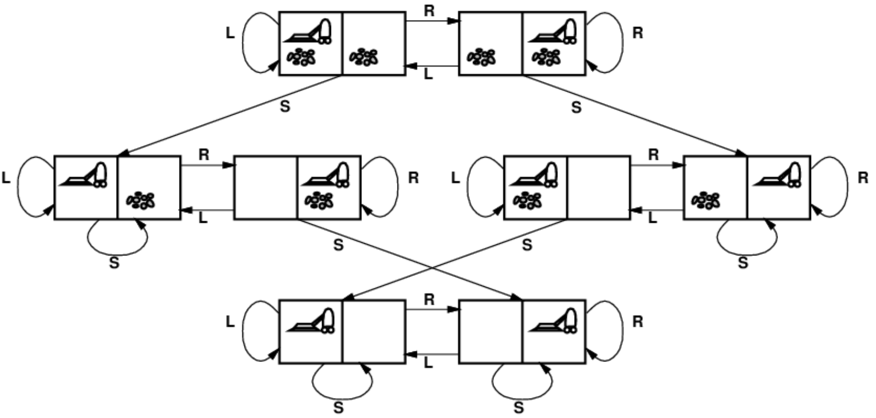

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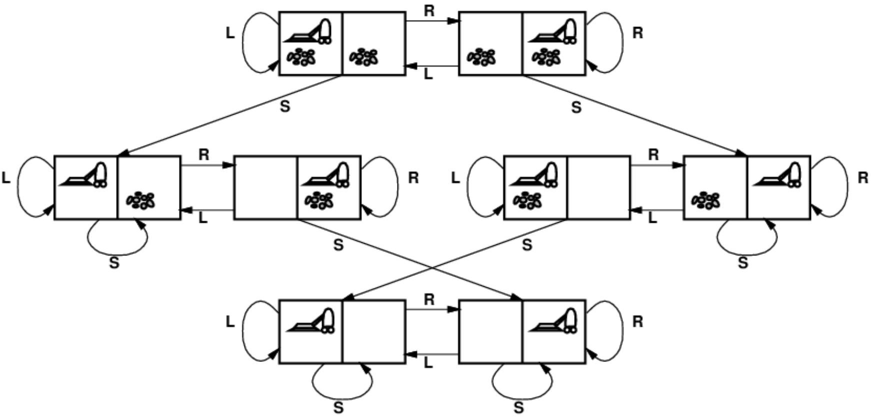

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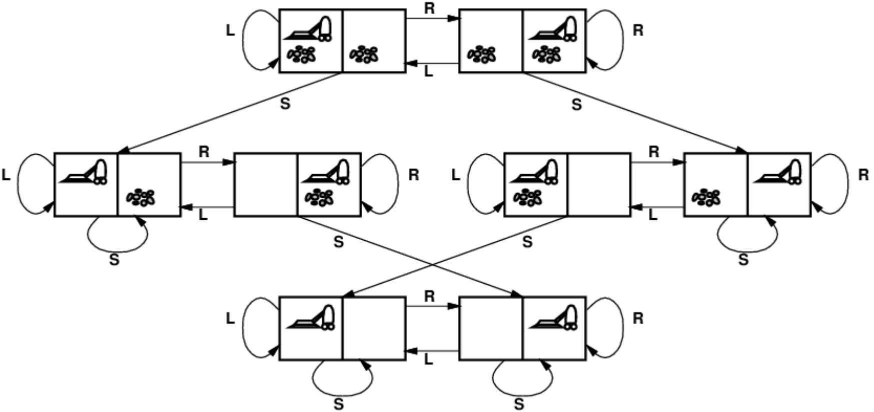

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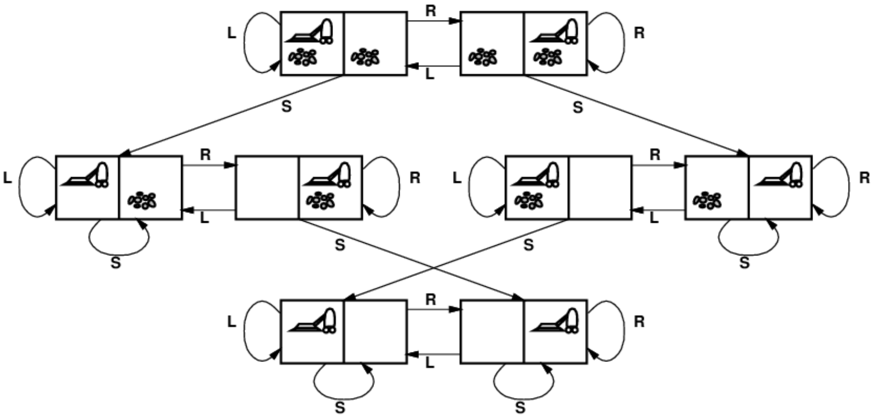

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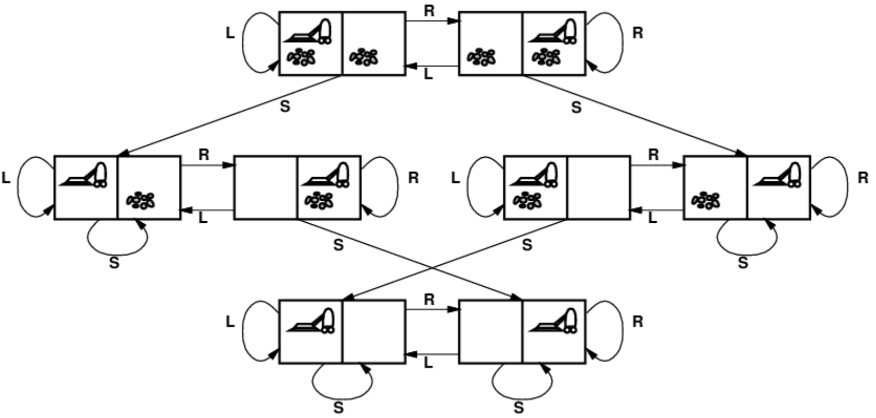

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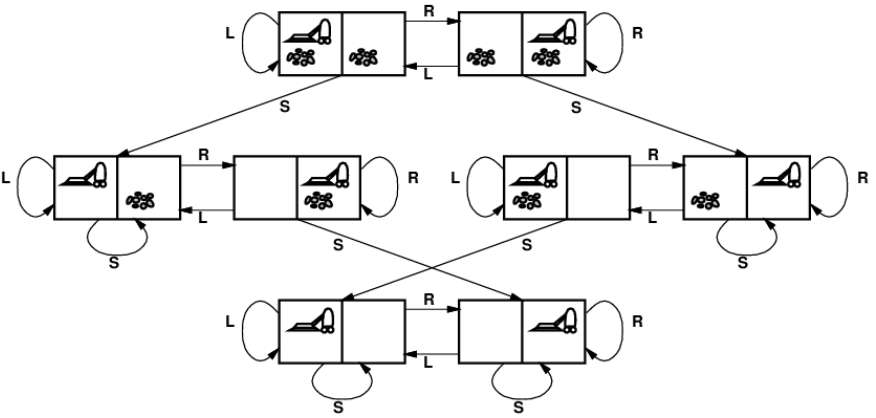

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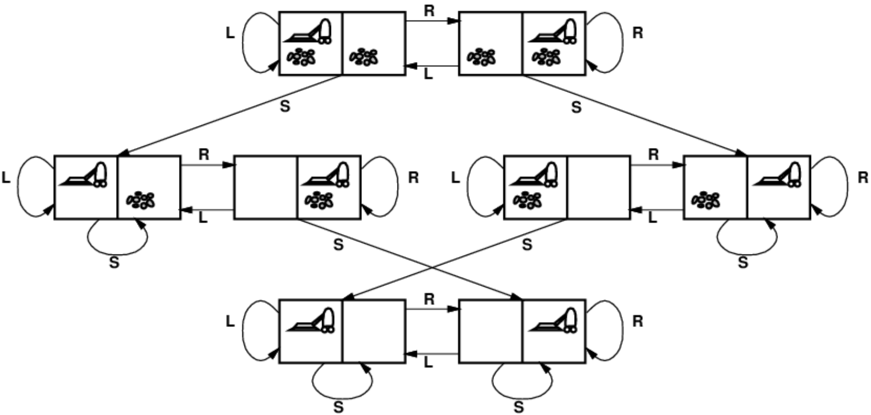

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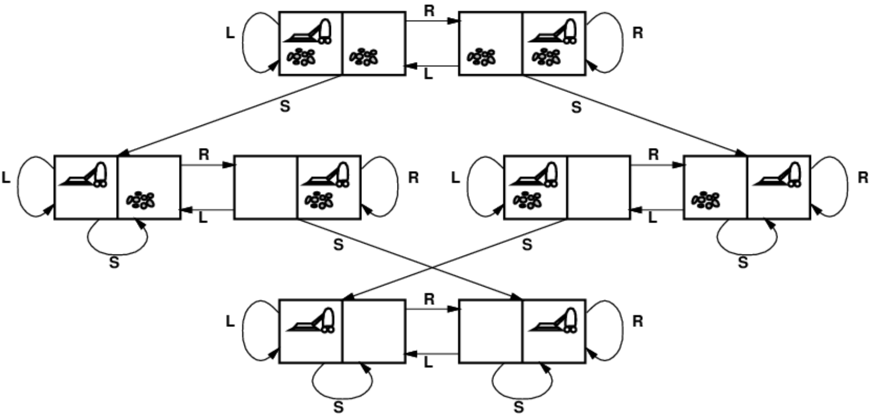

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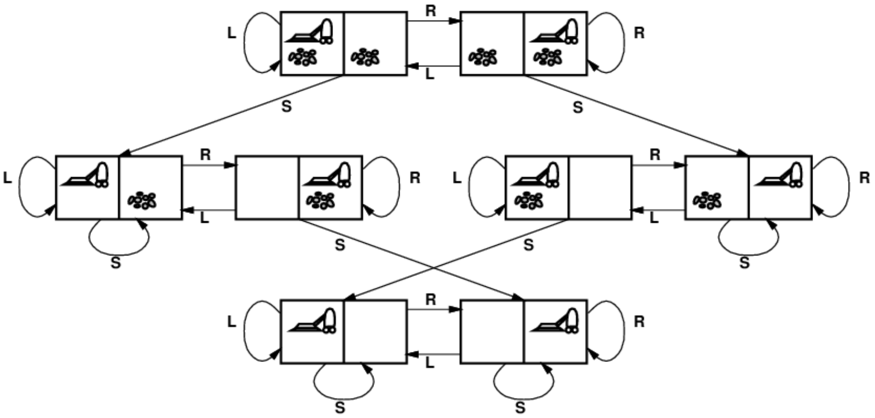

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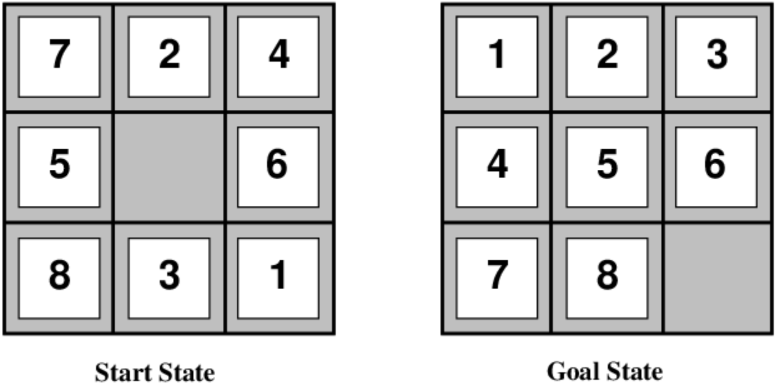

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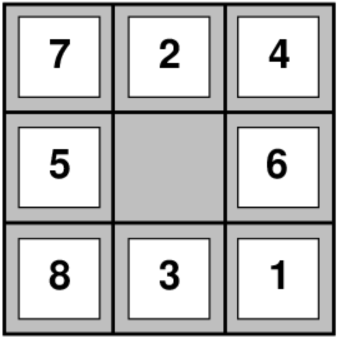
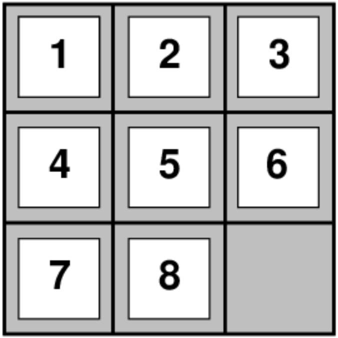


Start State



Goal State

states??: integer locations of tiles (ignore intermediate positions)

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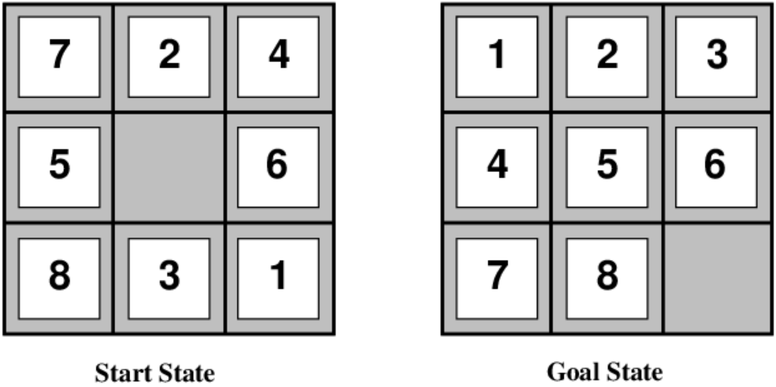

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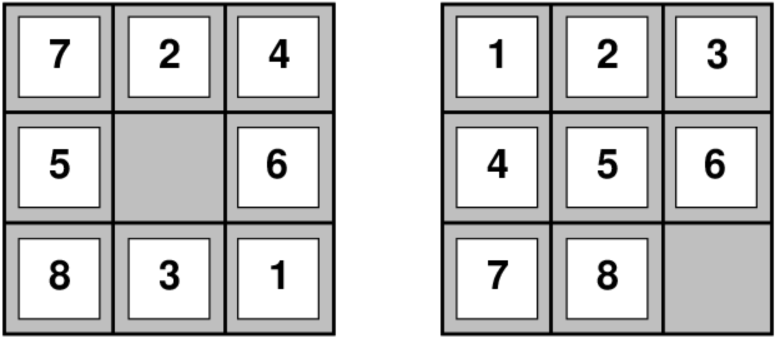

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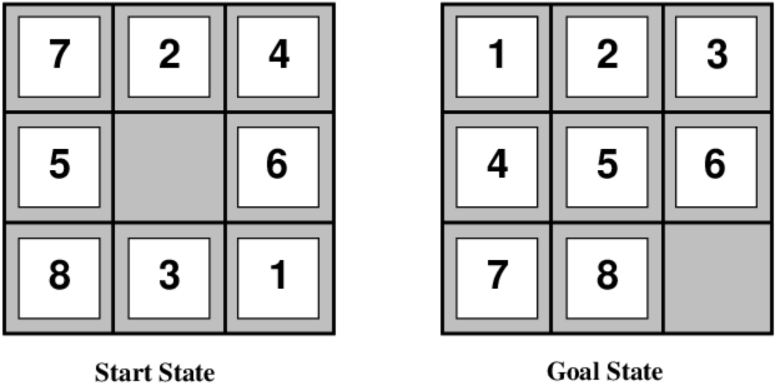

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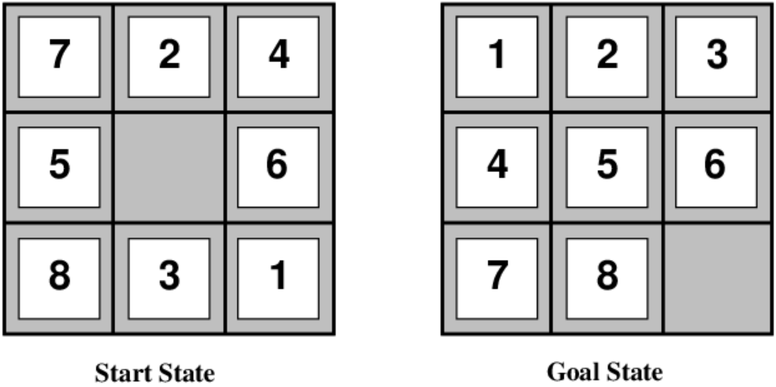

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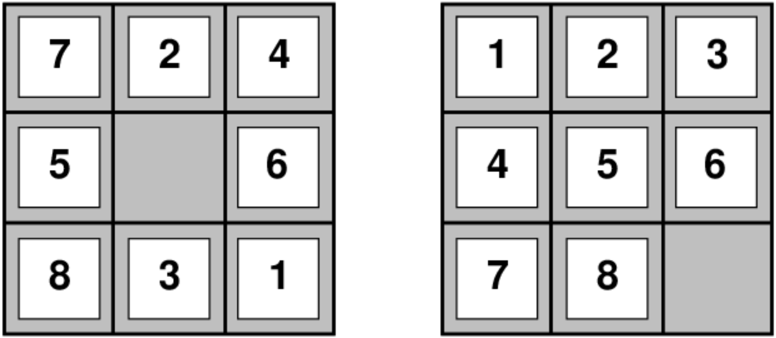

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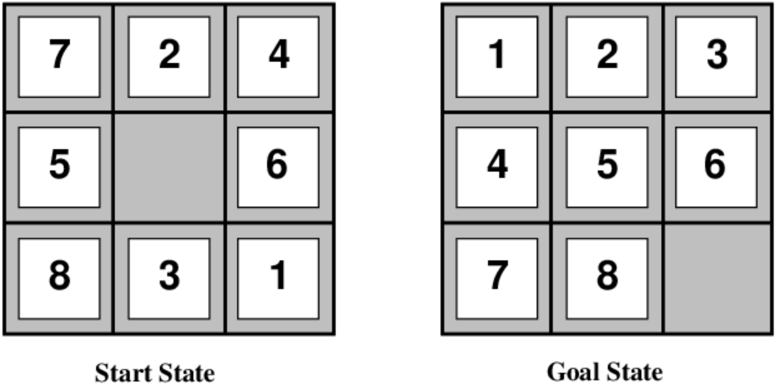

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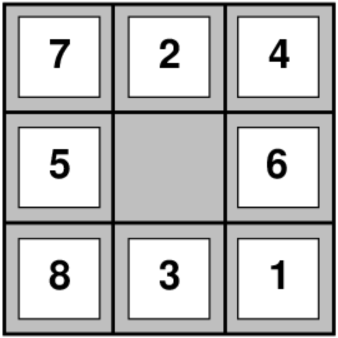

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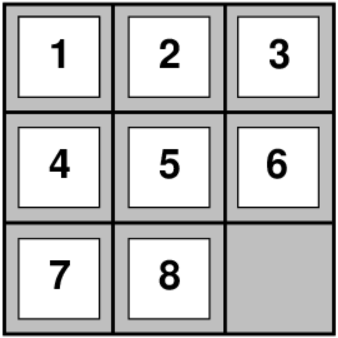

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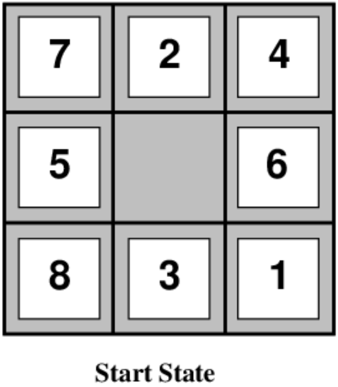
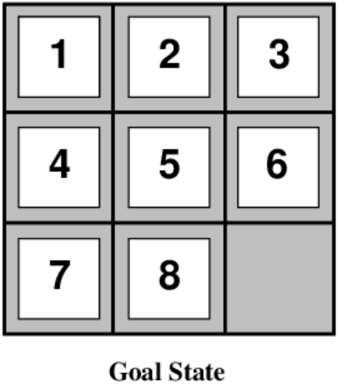
goal test??: = goal state (given)

Example: The 8-puzzle





states??: integer locations of tiles (ignore intermediate positions)

How many states?

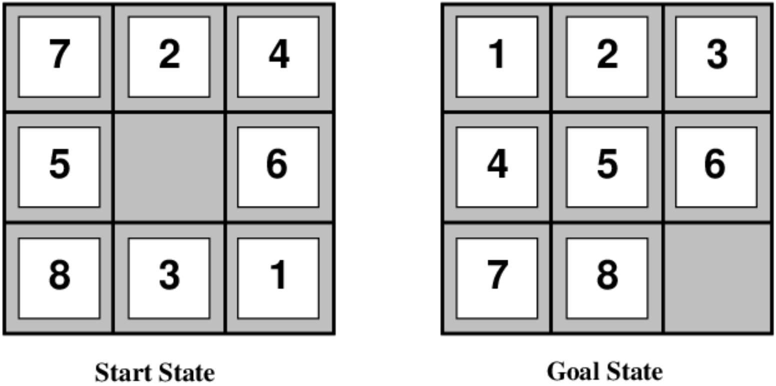
actions??: blank space "moves" Left, Right, Up, Down

transition model??: Given state and action, returns resulting state

goal test??: = goal state (given)

path cost??:

Example: The 8-puzzle



7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

states??: integer locations of tiles (ignore intermediate positions)

How many states?

actions??: blank space "moves" Left, Right, Up, Down

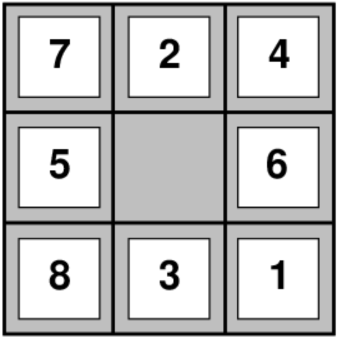
transition model??: Given state and action, returns resulting state

goal test??: = goal state (given)

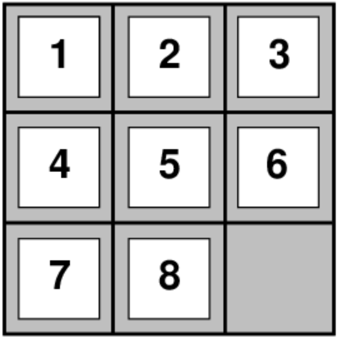
path cost??: 1 per move

[Note: optimal solution of *n*-Puzzle family is NP-hard!]

Example: The 8-puzzle



Start State



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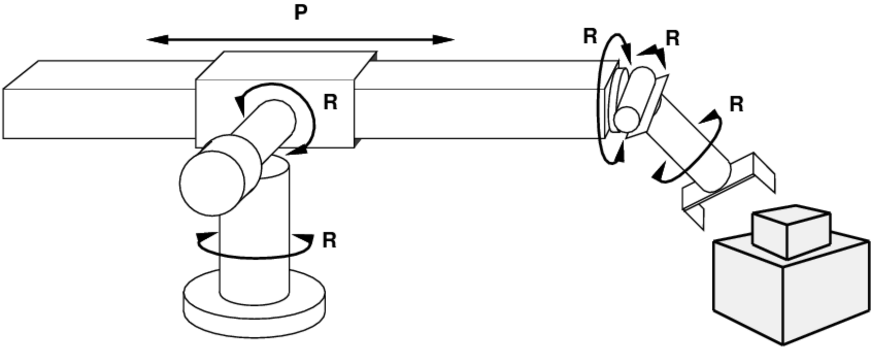
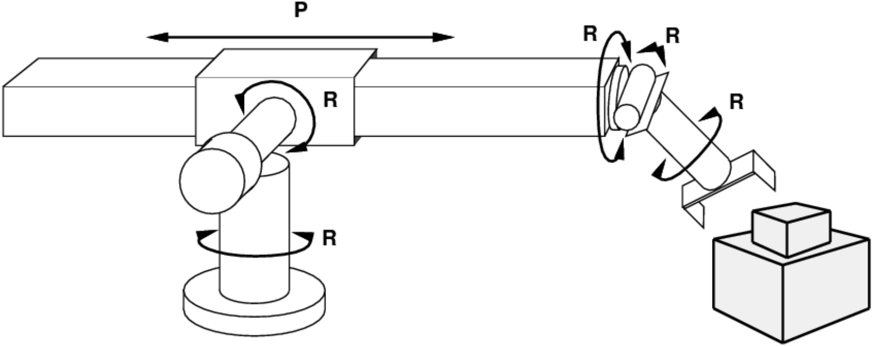
Example: Robotic Assembly



states??:

Example: Robotic Assembly



states??: real-valued coordinates of robot joint angles + parts of the object to be assembled

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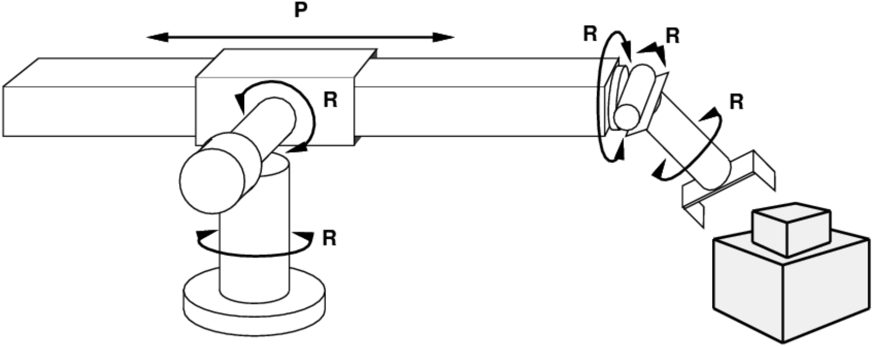

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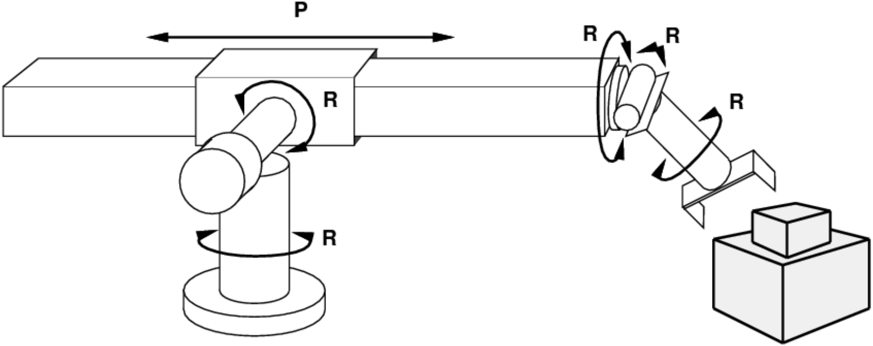

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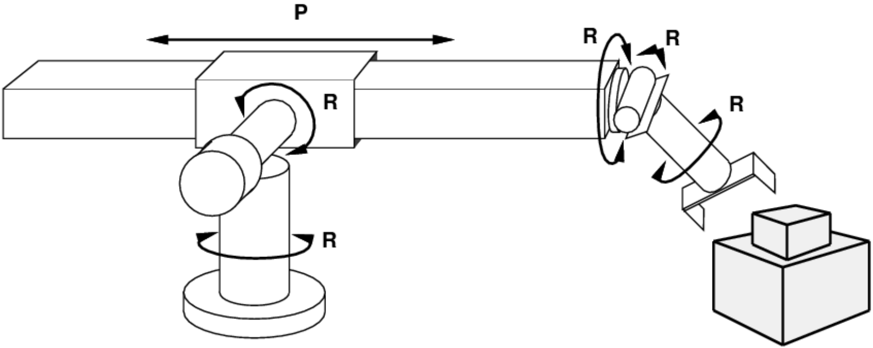

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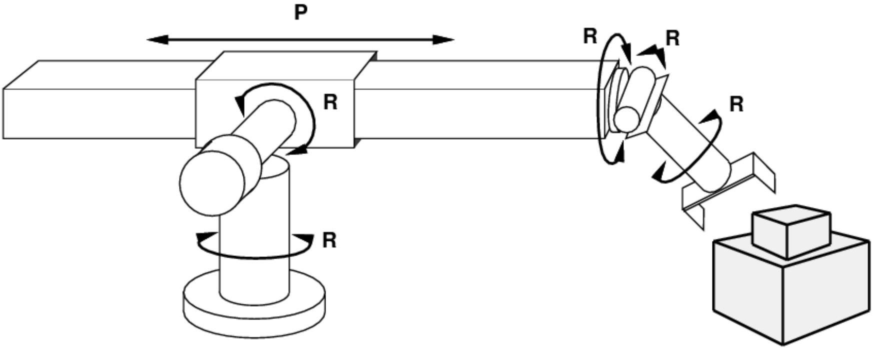

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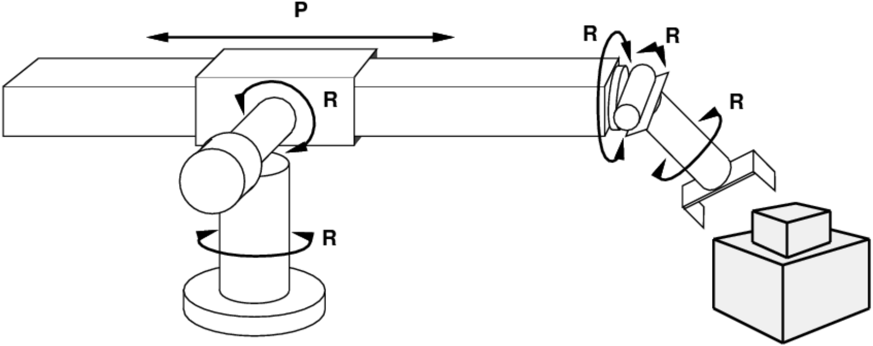
Example: Robotic Assembly



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actions??: continuous motions of robot joints
transition model??: state+action yields new state
goal test??:

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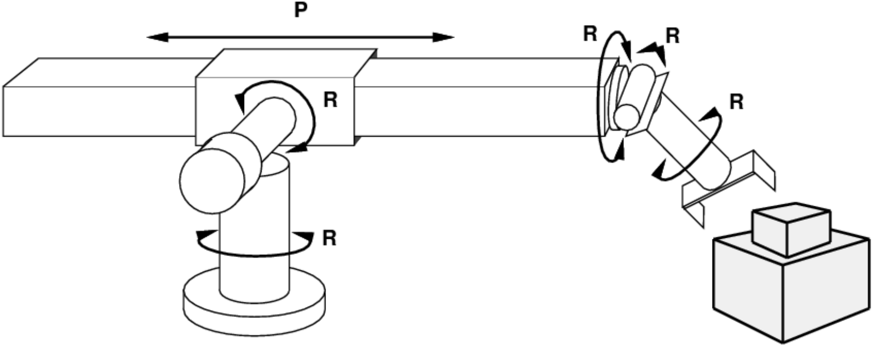

states??: real-valued coordinates of robot joint angles + parts of the object to be assembled
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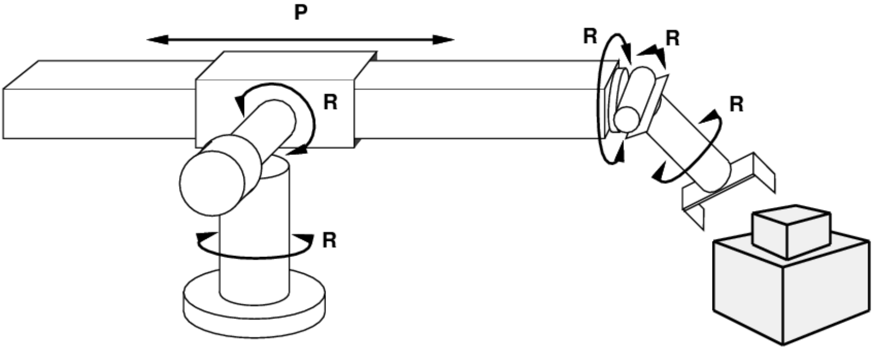
transition model??: state+action yields new state
goal test??: complete assembly **with no robot included!**

Example: Robotic Assembly



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path cost??:

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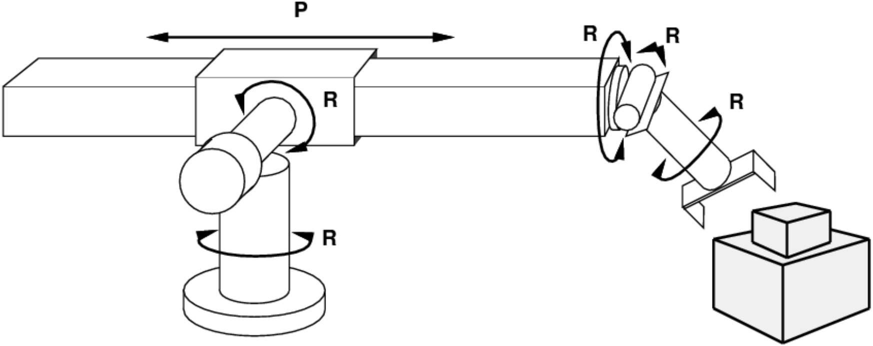

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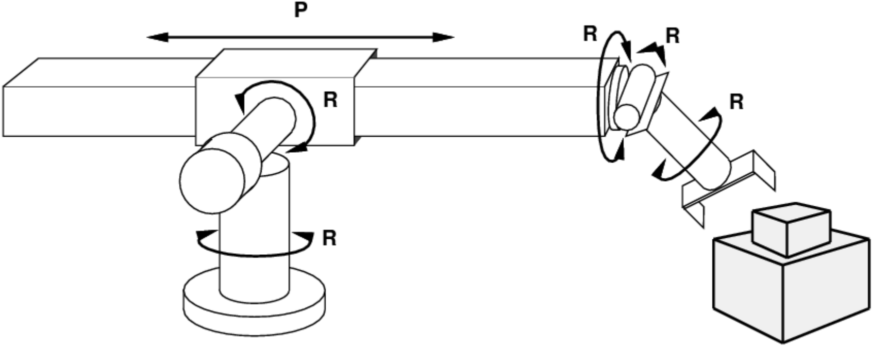

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Route-finding and Tour-finding Problems

The vacuum cleaner problem, 8-puzzle (block sliding), 8-queens, and others are examples of toy, route-finding problems.

Real-world route-finding problems can be found in robot navigation, manipulation, assembly, airline travel web-planning, and more.

Tour-finding problems are slighly different: "visit every city at least once, starting and ending in Bucharest."

Traveling salesperson problem (TSP): find shortest tour that visits each city exactly once, NP-hard.

Other related, complex problems: packing, scheduling, VLSI layout, protein folding, protein design.

Searching for Solutions

Choosing states and actions:

- abstraction: remove unnecessary information from representation; makes it cheaper to find a solution

Searching for Solutions:

- operators expand a state: generate new states from present ones
- fringe or frontier: discovered states to be expanded
- search strategy: tells which state in fringe set to expand next

Measuring Performance:

- does it find a solution?
- what is the search cost?
- what is the total cost (path cost + search cost)

Search Trees

A Search Tree:

A “what if” tree of plans and their outcomes

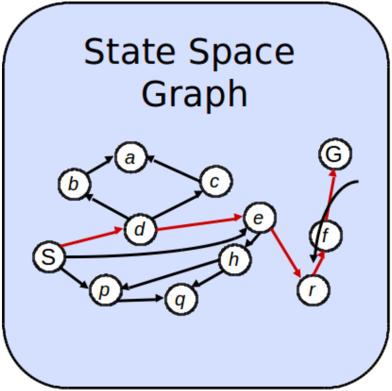
The start state is the root node

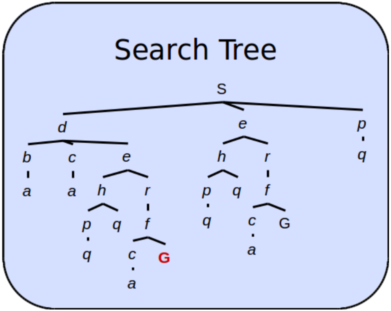
Children correspond to successors

Nodes show states, but correspond to PLANS that achieve those states

For most problems, we can never actually build the whole tree

State Space Graphs vs. Search Trees

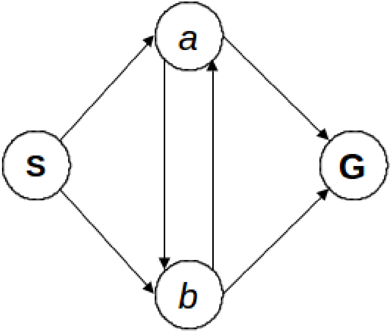




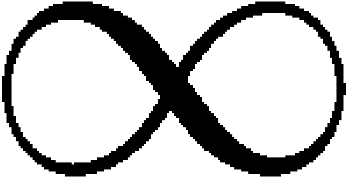
We construct both on demand and we construct as little as possible.

Quiz: State Space Graphs vs. Search Trees

Consider this 4-state space graph:



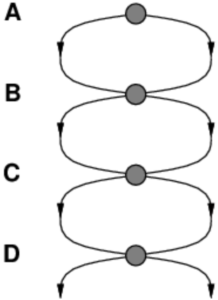
How big is it's search tree?

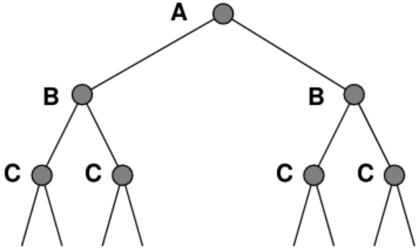


Lots of repeated structure in the search tree!

Repeated States

Failure to detect repeated states can turn a linear problem into an exponential one!

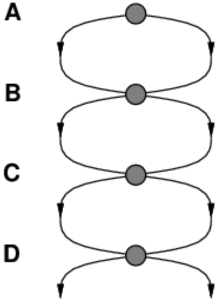


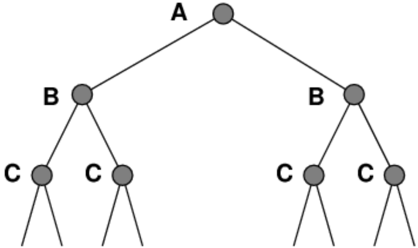


Repeated structure can be easily avoided:

Repeated States

Failure to detect repeated states can turn a linear problem into an exponential one!





Repeated structure can be easily avoided: How?

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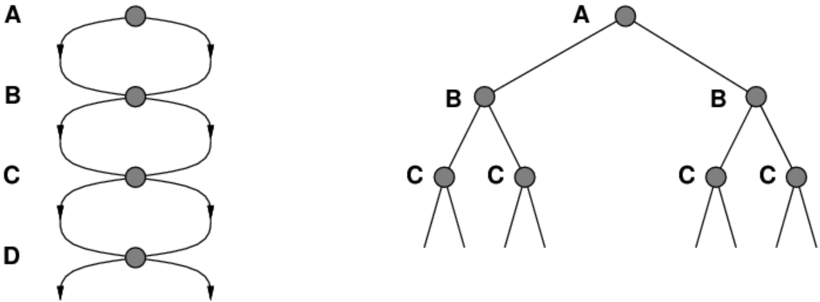


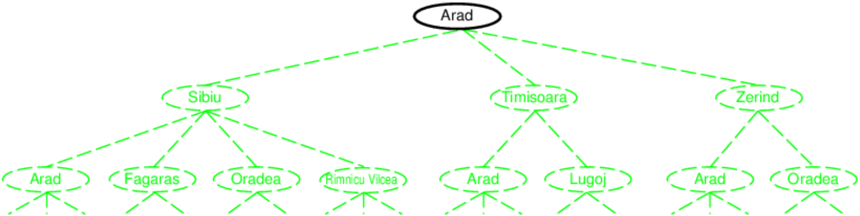
Repeated structure can be easily avoided: How?

Graph Search

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure

    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
    end
```

Searching with a Search Tree



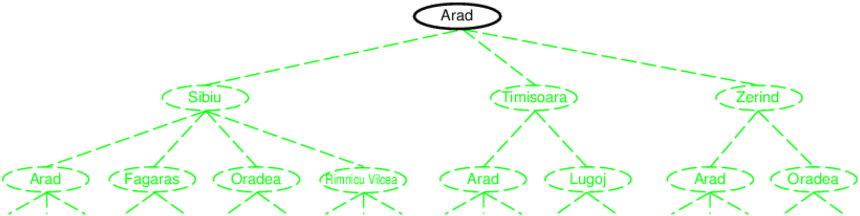
Basic idea:

Expand out potential plans (tree nodes)

Maintain a fringe of partial plans under consideration

Try to expand as few tree nodes as possible (Why?)

Searching with a Search Tree



Basic idea:

Expand out potential plans (tree nodes)

Maintain a fringe of partial plans under consideration

Try to expand as few tree nodes as possible (Why?)

(Discrete) Search Algorithms

Basic idea:
offline, simulated exploration of state space
by generating successors of already-explored states
(a.k.a. expanding states)

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

Fundamental Properties of Discrete Search Algorithms

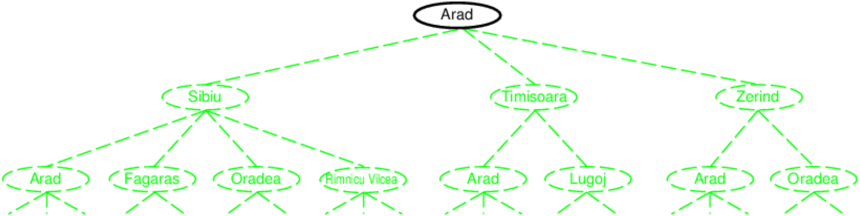
Fundamental to Graph Search/Traversal Algorithms:

- Successor function: generate successors/neighbors and distinguish a **goal** state from a **non-goal state**.

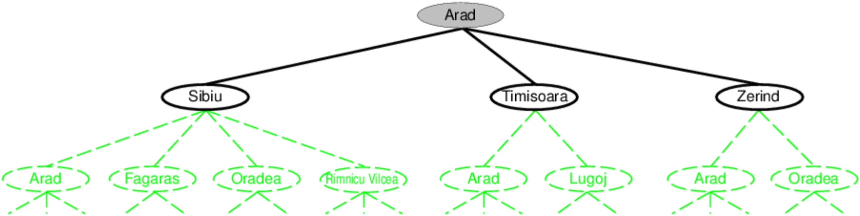
Completeness Goal should not be missed if a path exists.

Efficiency No edge should be traversed more than twice.

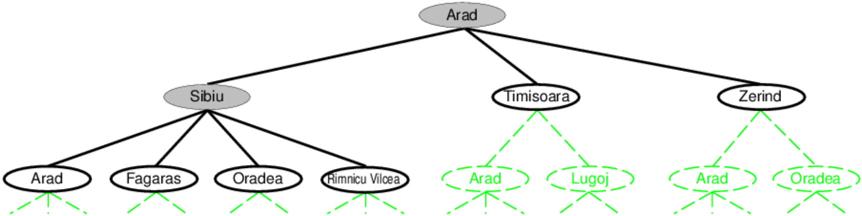
Tree Search Example



Tree Search Example



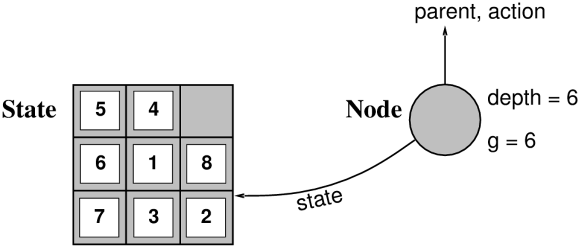
Tree Search Example



Implementation: States vs. Nodes

A state is a (representation of) a physical configuration
A node is a data structure constituting part of a search tree

includes **parent**, **children**, **depth**, **path cost** $g(x)$
States do not have parents, children, depth, or path cost!



The EXPAND function creates new nodes, filling in the various fields and using the SUCCESSORFN of the problem to create the corresponding states.

General Tree Search

- **Important insight:**
  - Any search algorithm constructs a tree, adding to it vertices from state-space graph G in some order
  - $G = (V, E)$ —— look at it as split in two: set S on one side and $V - S$ on the other
  - search proceeds as vertices are taken from $V - S$ and added to S
  - search ends when $V - S$ is empty or goal found
  - First vertex to be taken from $V - S$ and added to S?
  - Next vertex? (... expansion ...)
  - Where to keep track of these vertices? (... fringe/frontier ...)
- **Important ideas:**
  - Fringe (frontier into $V - S$/border between S and $V - S$)
  - Expansion (neighbor generation so can add to fringe)
  - Exploration strategy (what order to grow S?)
- **Main question:**
  - which fringe/frontier nodes to explore/expand next?
  - strategy distinguishes search algorithms from one another

Implementation: General Tree Search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE(node)) then return node
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

```
function EXPAND( node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in SUCCESSOR-FN(problem, STATE[node]) do
        s ← a new NODE
        PARENT-NODE[s] ← node;  ACTION[s] ← action;  STATE[s] ← result
        PATH-COST[s] ← PATH-COST[node] + STEP-COST(STATE[node], action, result)
        DEPTH[s] ← DEPTH[node] + 1
        add s to successors
    return successors
```

Search Strategies

A strategy is defined by picking the **order of node expansion**

Strategies are evaluated along the following dimensions:

- completeness—does it always find a solution if one exists?
- time complexity—number of nodes generated/expanded
- space complexity—maximum number of nodes in memory
- optimality—does it always find a least-cost solution?

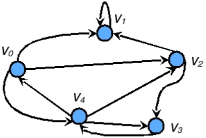
Time and space complexity are measured in terms of:

- b—maximum branching factor of the search tree
- d—depth of the least-cost solution
- m—maximum depth of the state space (may be ∞)

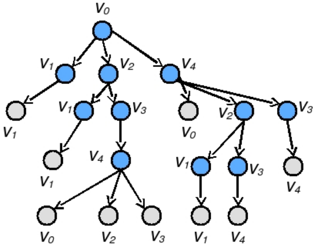
Uninformed Graph Search

Characteristics of Uninformed Graph Search/Traversal:

- There is no additional information about states/vertices beyond what is provided in the problem definition.
- All that the search does is generate successors/neighbors and distinguish a **goal** state from a **non-goal state**.



The systematic search "lays out" all paths from initial vertex; it traverses the search tree of the graph.



Uninformed Graph Search

F: search data structure (fringe)
parent array: stores "edge comes from" to record visited states

```
F.insert(v)
parent[v] ← true
while not F.isEmpty do
    u ← F.extract()
    if isGoal(u) then
        return true
    for each v in outEdges(u) do
        if no parent[v] then
            F.insert(v)
            parent[v] ← u
```

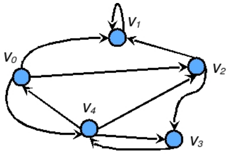


Figure: Graph

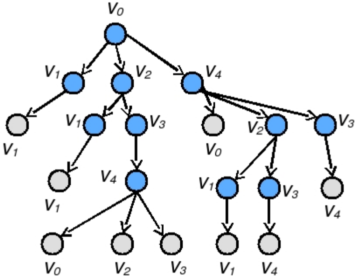
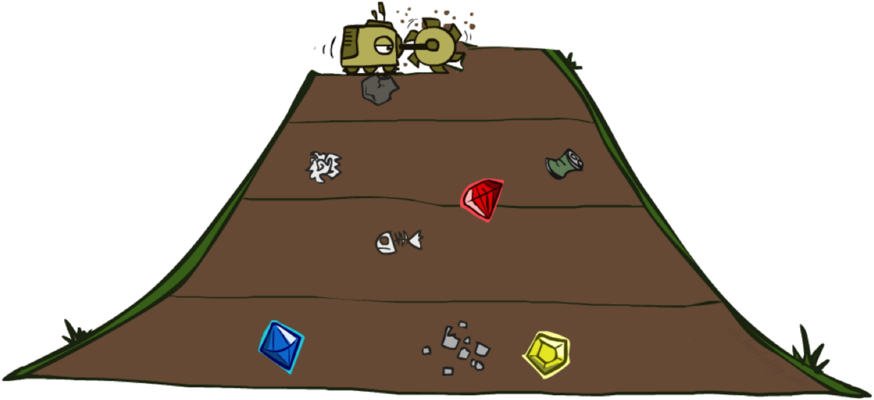


Figure: Search Tree of Graph

Uninformed Search Algorithms

- Breadth-first Search (BFS)
- Depth-first Search (DFS)
- Depth-limited search (DLS)
- Iterative Deepening Search (IDS)

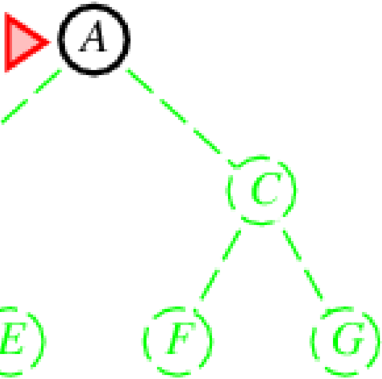
Breadth-first Search (BFS)



Breadth-first Search (BFS)

Strategy: Expand shallowest unexpanded node

Implementation:
fringe = first-in first-out (FIFO), i.e., unvisited successors go at end
F is a queue



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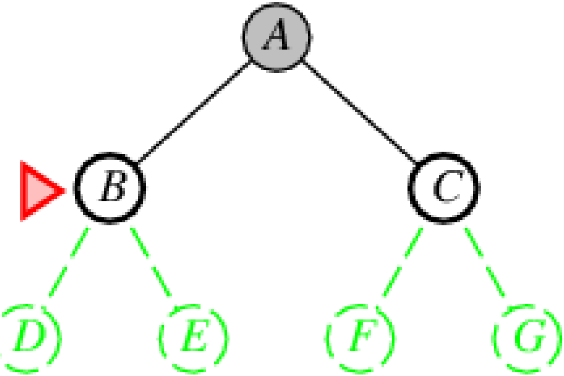

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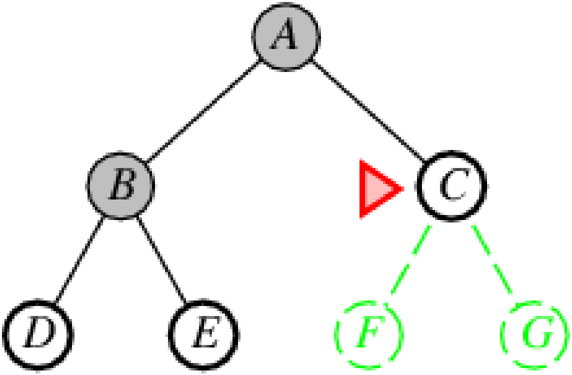

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Strategy: **Expand shallowest unexpanded node**

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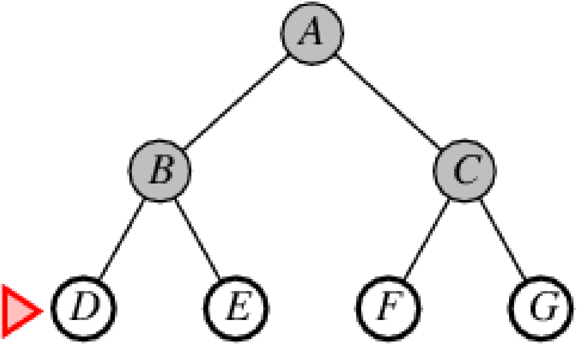

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F: search data structure (fringe)

F is a queue (FIFO) in BFS!

parent array: stores "edge comes from" to record visited states

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        if no parent[v] then
            F.insert(v)
            parent[v] ← u
```

Running Time?

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F is a queue (FIFO) in BFS!

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Running Time?

Let V and E be vertices and edges in search tree

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Running Time?

Let V and E be vertices and edges in search tree

$O(|V| + |E|)$

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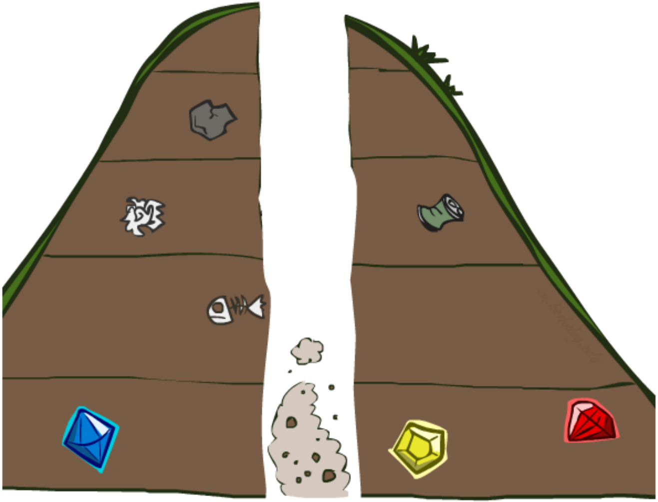

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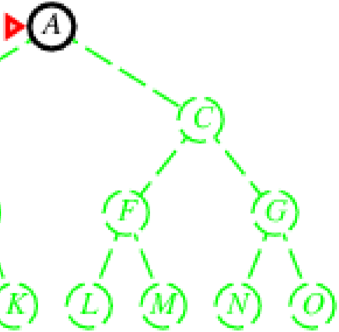

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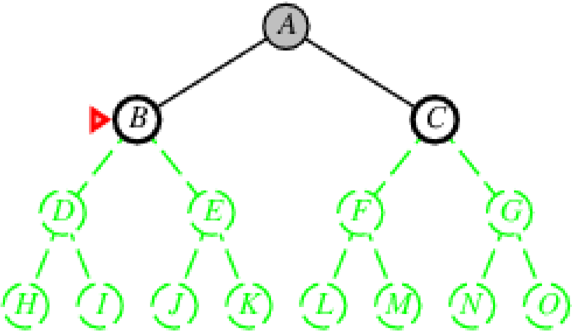

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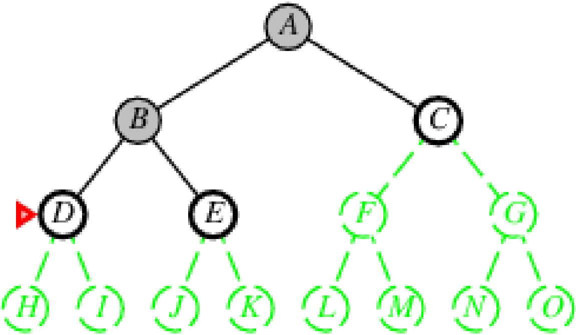

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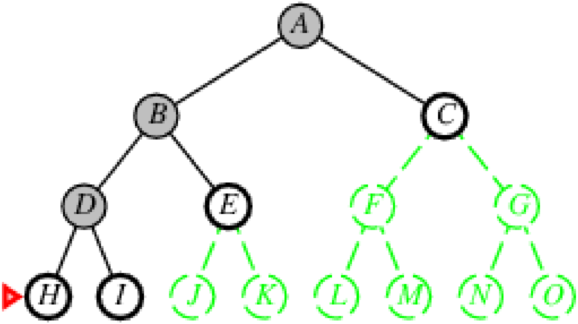

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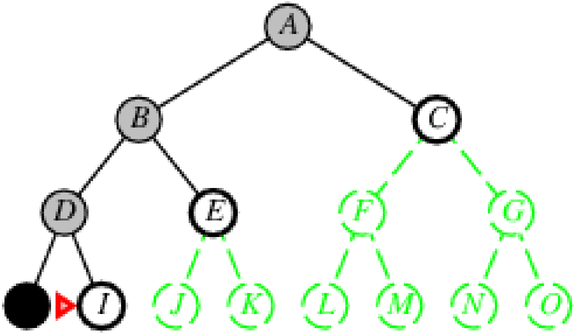

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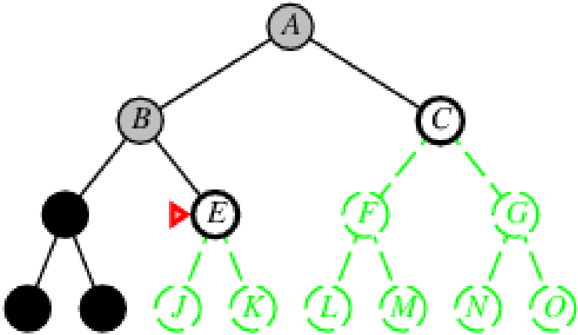

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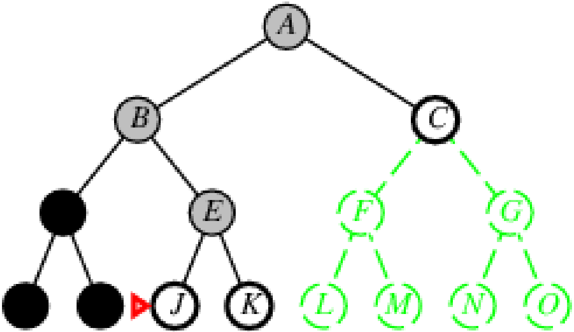

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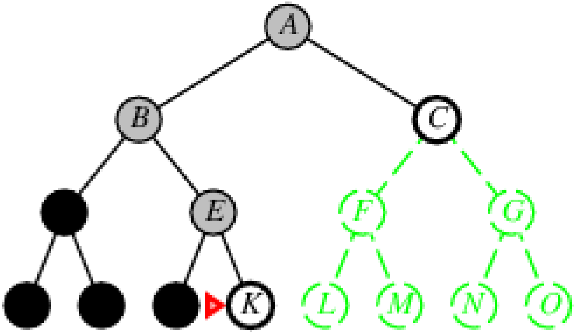

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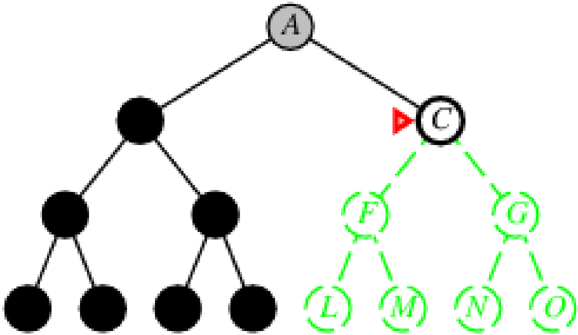

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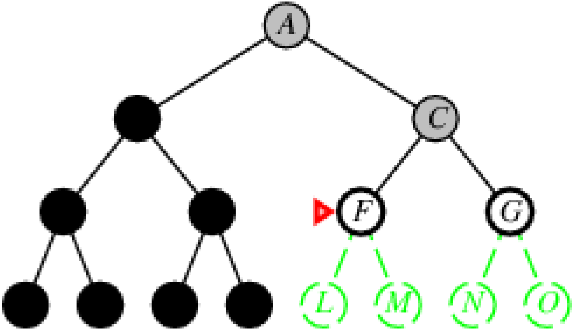

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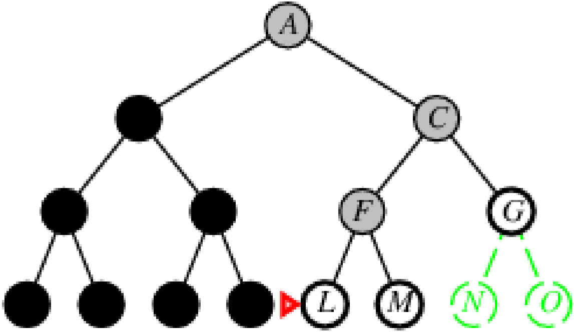

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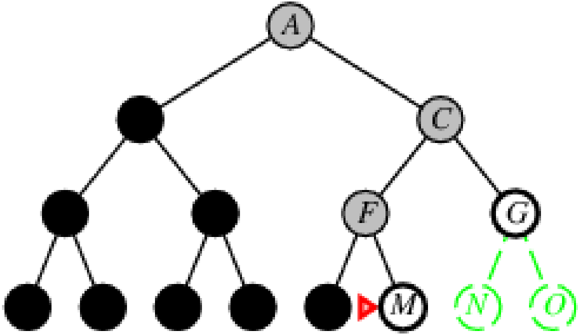

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BFS vs. DFS

- When will BFS outperform DFS?
- When will DFS outperform BFS?

Another Advantage of DFS

```
RecursiveDFS($v$)
if $v$ is unmarked then
    mark $v$
    for each edge $v, u$ do
        RecursiveDFS($u$)
```


Undiscovered
Unfinished
Active
Finished

Color arrays can be kept to indicate that a vertex is undiscovered, the first time it is discovered, when its neighbors are in the process of being considered, and when all its neighbors have been considered.

DFS can be used to timestamp vertices with when they are discovered and when they are finished. These start and finish times are useful in various applications of DFS regarding constraint satisfaction.

Depth-limited Search (DLS)

- Problem with DFS is presence of infinite paths
- DLS limits the depth of a path in search tree of DFS
- Modifies *DFS* by using a predetermined depth limit $\mathrm{d_l}$
- DLS is incomplete if the shallowest goal is beyond the depth limit d_l
- DLS is not optimal if $d < d_l$
- Time complexity is $O(b^{d_l})$ and space complexity is $O(b \cdot d_l)$

Depth-limited Search (DLS)

= DFS with depth limit d_l [i.e., nodes at depth d_l are not expanded]

Recursive implementation:

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
    RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if GOAL-TEST(problem, STATE[node]) then return node
    else if DEPTH[node] = limit then return cutoff
    else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
```

Iterative Deepening Search (IDS)

- Finds the best depth limit by incrementing d_l until goal is found at $d_l = d$
- Can be viewed as running DLS with consecutive values of d_l
- IDS combines the benefits of both DFS and BFS
- Like DFS, its space complexity is $O(b \cdot d)$
- Like BFS, it is complete when the branching factor is finite, and it is optimal if the path cost is a non-decreasing function of the depth of the goal node
- Its time complexity is $O(b^d)$
- IDS is the preferred uninformed search when the state space is large, and the depth of the solution is not known

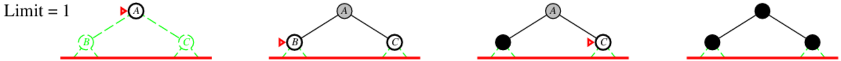
Iterative Deepening Search (IDS)

```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution
    inputs: problem, a problem

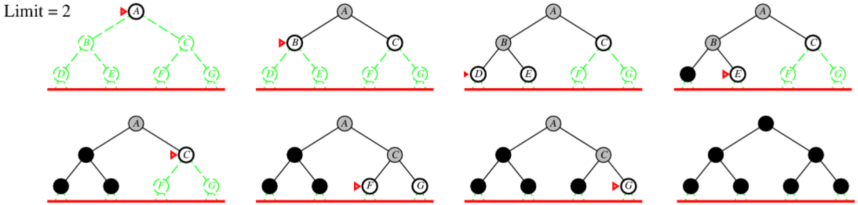
    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
    end
```

Iterative Deepening Search (IDS) @ $d_l = 0$

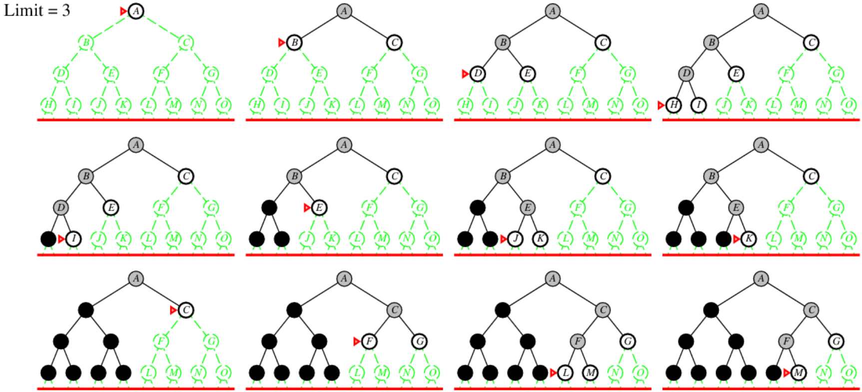
Iterative Deepening Search (IDS) @ $d_l = 1$



Iterative Deepening Search (IDS) @ $d_l = 2$



Iterative Deepening Search (IDS) @ $d_l = 3$



Summary of Uninformed Search Algorithms

Criterion	Breadth-First	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	No	Yes, if $d_l \geq d$	Yes
Time	b^{d+1}	b^m	b^{d_l}	b^d
Space	b^{d+1}	bm	bd_l	bd
Optimal?	Yes*	No	No	Yes*

Uninformed Search Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- IDS uses only linear space and not much more time than other uninformed algorithms
- Graph search can be exponentially more efficient than tree search
- What about least-cost paths with non-uniform state-state costs?
    - That is the subject of next lecture