

CS 485: Autonomous Robotics

Sampling-Based Motion Planning

Amarda

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George Mason University

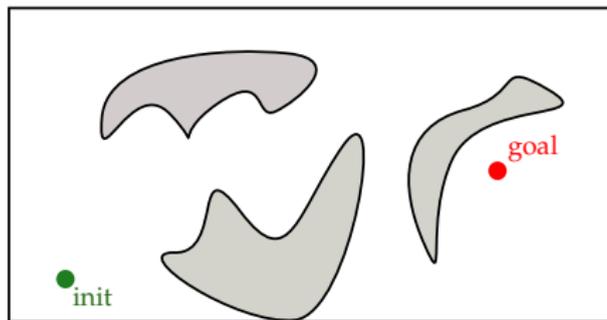
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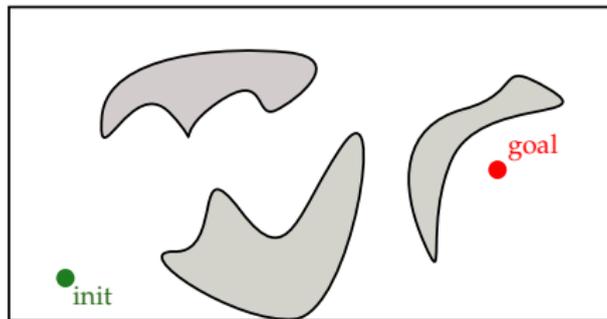
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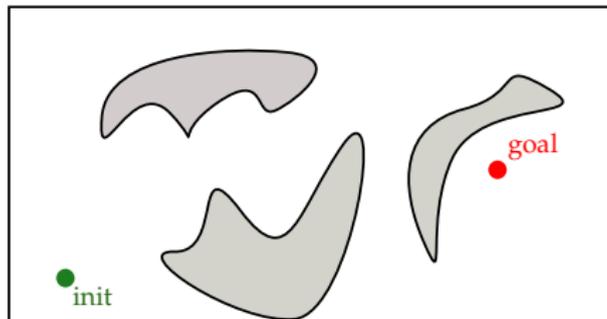


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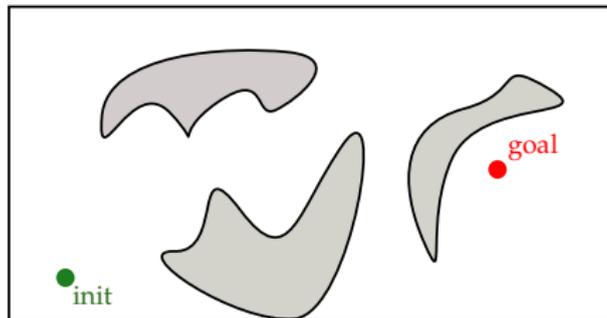
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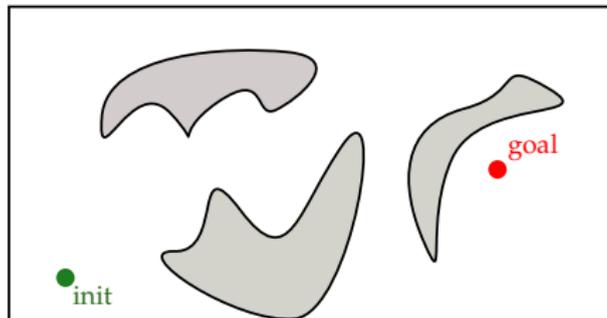
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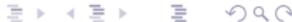
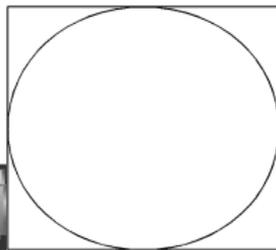
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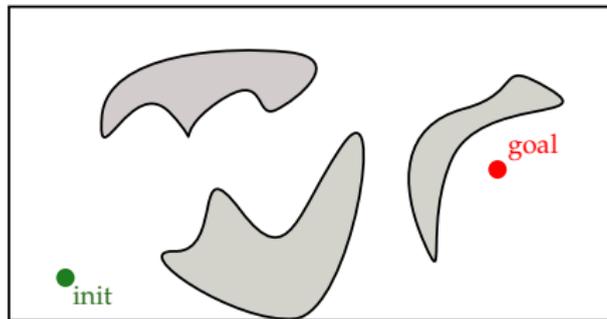
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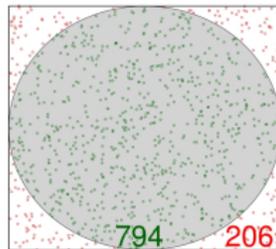
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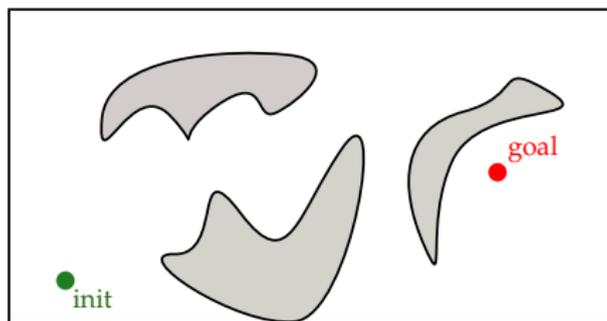


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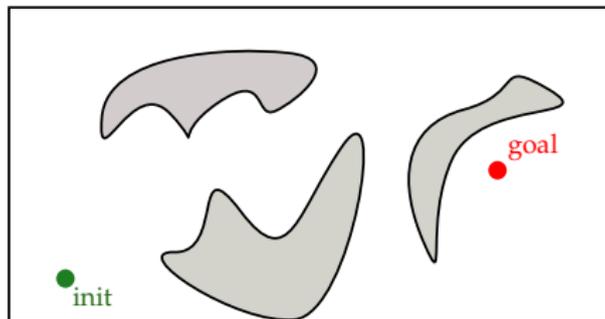


Monte-Carlo Idea:

- Define input space
- Generate inputs at random by *sampling* the input space
- Perform a deterministic computation using the input samples
- Aggregate the partial results into final result

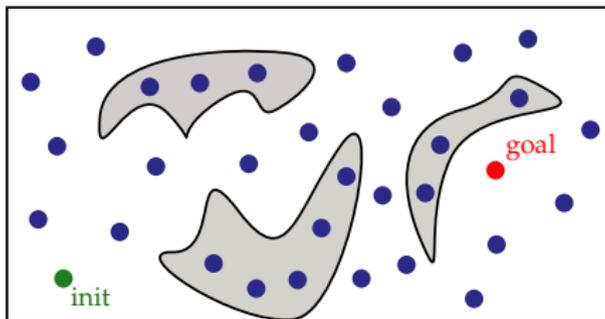
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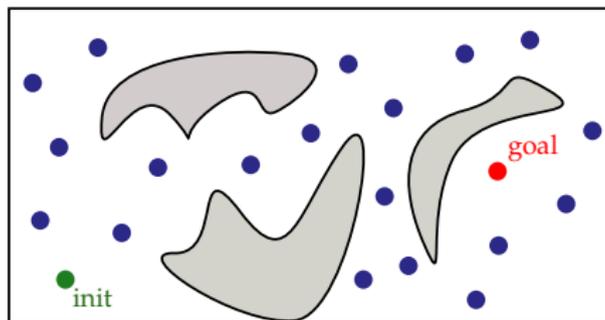
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- **Sample points**

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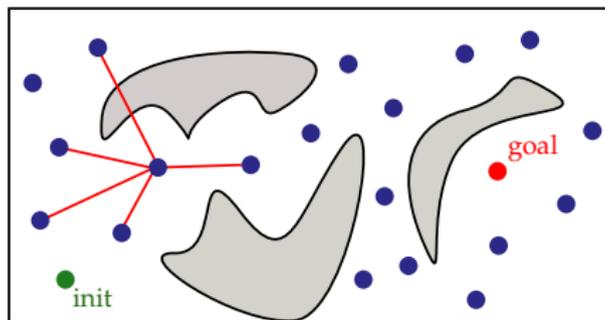
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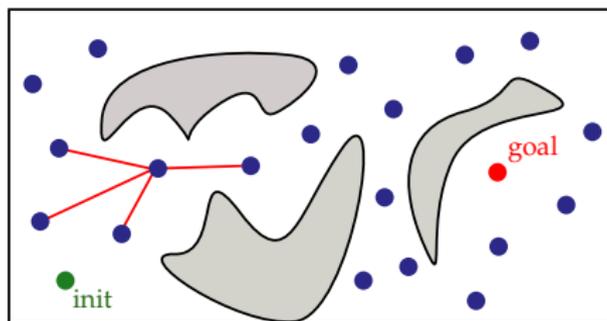
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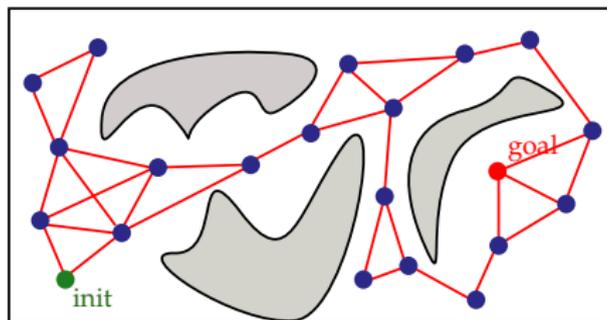
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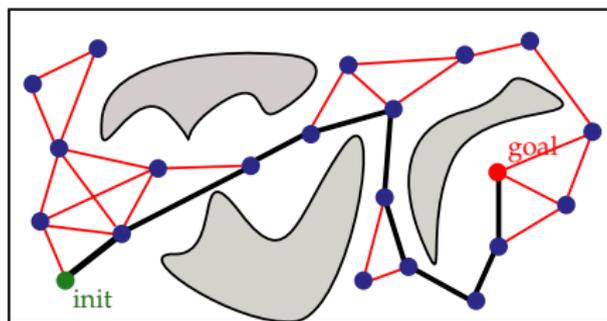
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- ⇒ Gives rise to a graph, called the *roadmap*
- ⇒ Collision-free path can be found by performing graph search on the roadmap

Probabilistic RoadMap (PRM) Method

[Kavraki, Švestka, Latombe, Overmars 1996]

0. Initialization

add q_{init} and q_{goal} to roadmap vertex set V

1. Sampling

repeat several times

$q \leftarrow \text{SAMPLE}()$

if $\text{ISCOLLISIONFREE}(q) = \text{true}$

add q to roadmap vertex set V

2. Connect Samples

for each pair of neighboring samples $(q_a, q_b) \in V \times V$

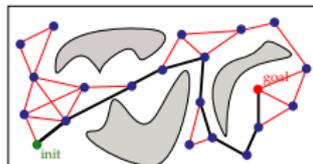
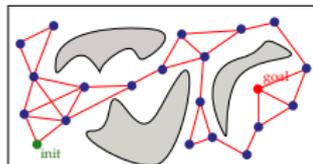
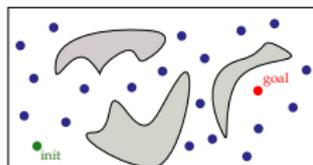
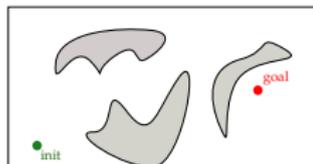
path $\leftarrow \text{GENERATELOCALPATH}(q_a, q_b)$

if $\text{ISCOLLISIONFREE}(\text{path}) = \text{true}$

add (q_a, q_b) to roadmap edge set E

3. Graph Search

search graph (V, E) for path from q_{init} to q_{goal}



Sampling-based Path Planning

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- Computationally efficient
- Solves high-dimensional problems (with hundreds of DOFs)
- Easy to implement
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It offers *probabilistic completeness*

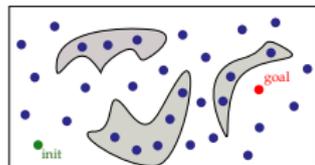
- When a solution exists, a probabilistically complete planner finds a solution with probability as time goes to infinity.
- When a solution does not exist, a probabilistically complete planner may not be able to determine that a solution does not exist.

PRM Applied to 2D-point Robot

$q = (x, y) \leftarrow \text{SAMPLE}()$

■ $x \leftarrow \text{RAND}(\min_x, \max_x)$

■ $y \leftarrow \text{RAND}(\min_y, \max_y)$



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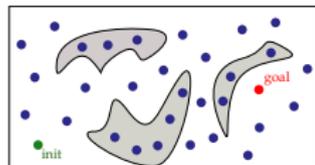
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■ Point inside/outside polygon test



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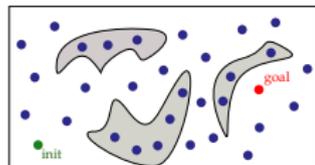
■ $y \leftarrow \text{RAND}(\min_y, \max_y)$

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$\text{path} \leftarrow \text{GENERATELOCALPATH}(q_a, q_b)$

■ Straight-line segment from point q_a to point q_b



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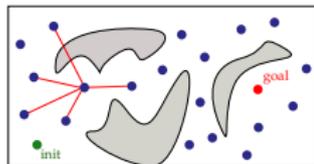
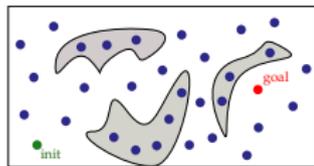
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$\text{path} \leftarrow \text{GENERATELOCALPATH}(q_a, q_b)$

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■ Segment-polygon intersection test

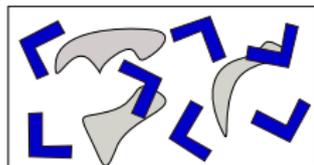


PRM Applied to 2D Rigid-Body Robot

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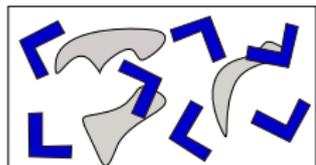
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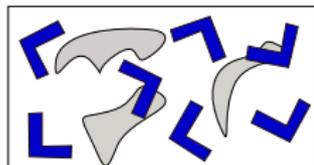
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$\text{ISSAMPLECOLLISIONFREE}(q)$

- Place rigid body in position and orientation specified by q
- Polygon-polygon intersection test

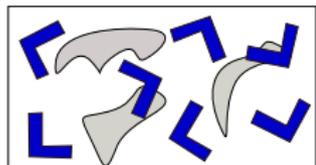
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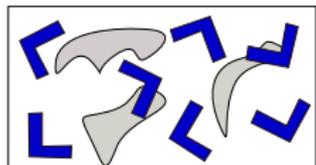
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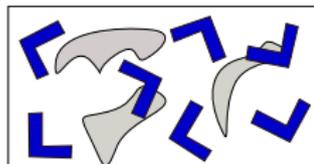
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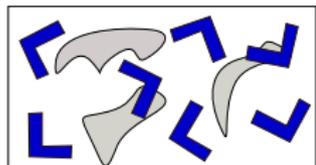
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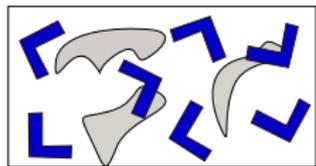
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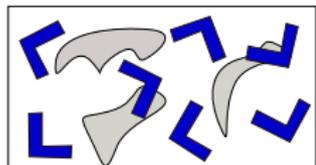
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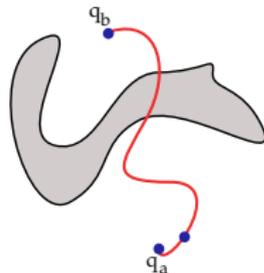
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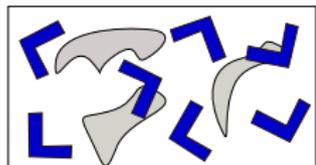
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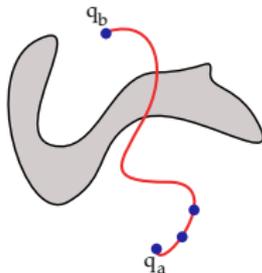
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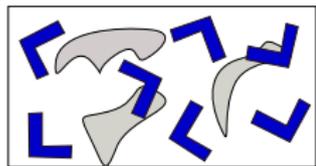
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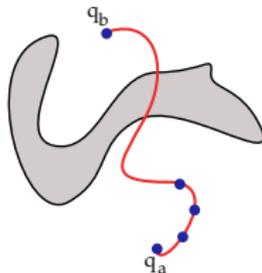
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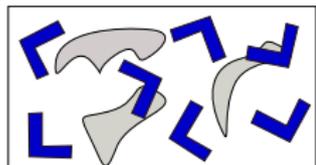
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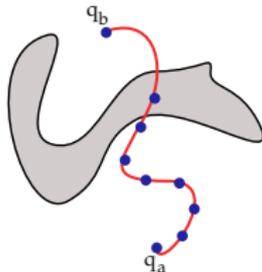
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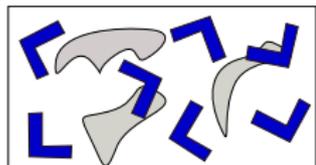
$\text{ISPATHCOLLISIONFREE}(\text{path})$

- Incremental approach



$q = (x, y, \theta) \leftarrow \text{SAMPLE}()$

- $x \leftarrow \text{RAND}(\min_x, \max_x)$; $y \leftarrow \text{RAND}(\min_y, \max_y)$;
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$\text{ISSAMPLECOLLISIONFREE}(q)$

- Place rigid body in position and orientation specified by q
- Polygon-polygon intersection test

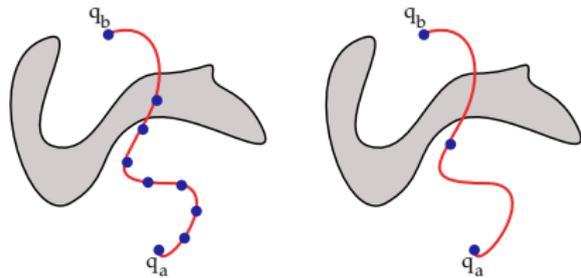
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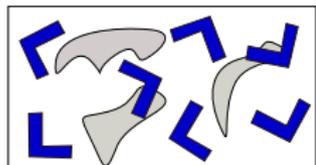
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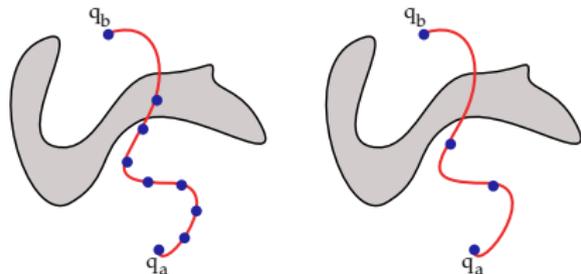
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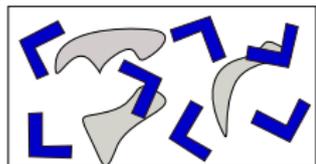
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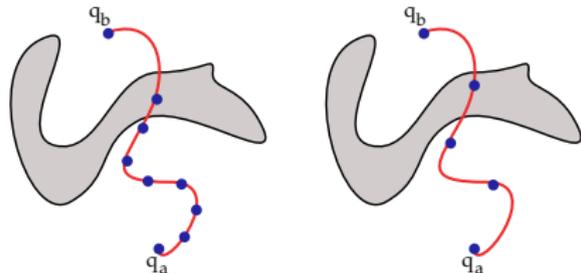
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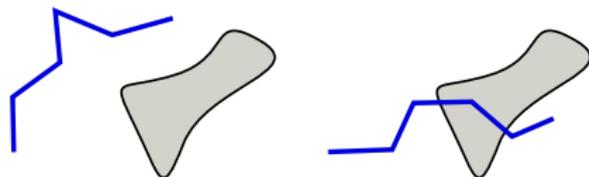
[piano] [manocha] [kcar] [tri] [buggy]



PRM Applied to Articulated Chain

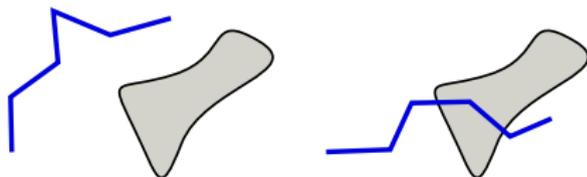
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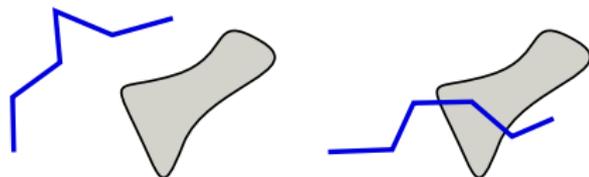


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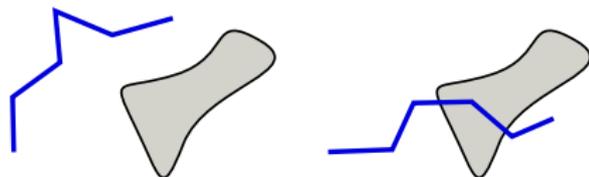
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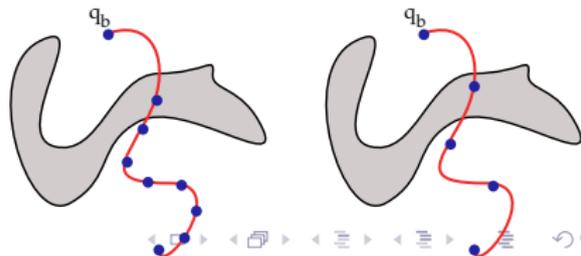
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Path Smoothing

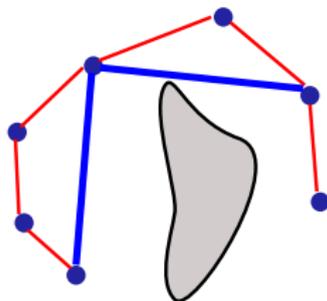
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- 1: **for** several times **do**
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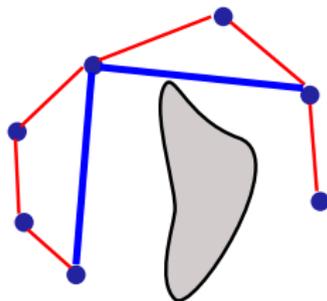


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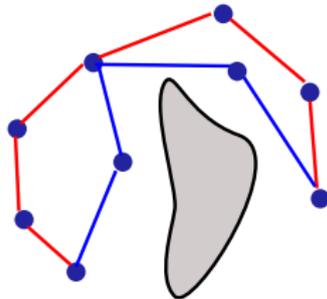
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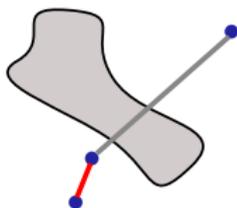
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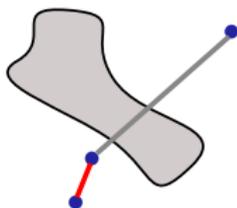
Connecting Roadmap Nodes to Nearest Neighbors

Edges between neighboring nodes are more likely to be collision free than edges between far away nodes



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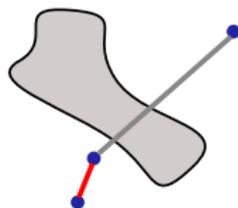
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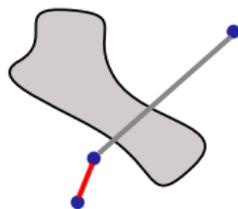
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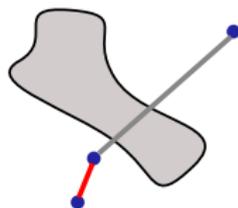
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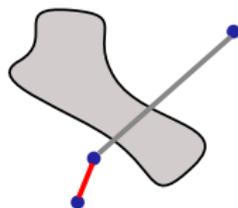


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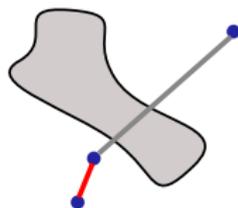
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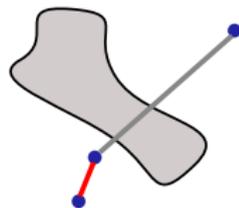
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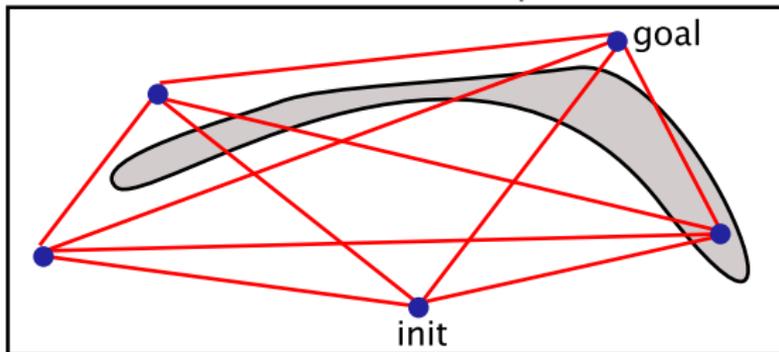
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- Computational challenges of nearest neighbors in high-dimensional spaces
 - Efficiency deteriorates rapidly
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- Alternative approach is to compute *approximate* nearest neighbors
 - [Plaku, Kavraki: WAFR 2006, SDM 2007]
 - Minimal losses in accuracy of neighbors
 - No loss in accuracy of overall path planner
 - Significant computational gains

Perform collision checking only when necessary

[Bohlin, Kavraki: Handbook on Randomized Computing 2000]

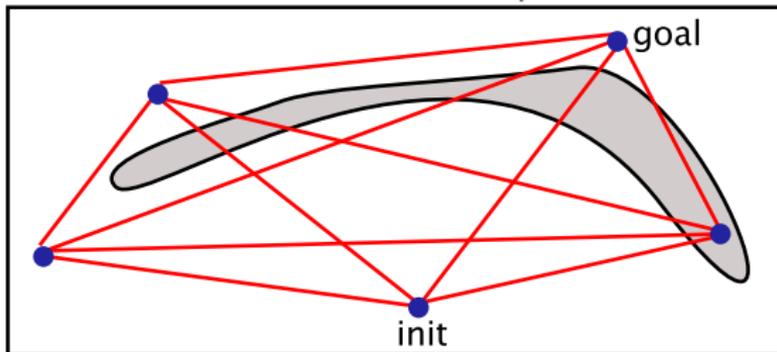


LAZYROADMAPCONSTRUCTION

- 1: $V \leftarrow V \cup \{q_{init}, q_{goal}\}; E \leftarrow \emptyset$
- 2: **for** several times **do**
- 3: $q \leftarrow$ generate config uniformly at random; $q.checked \leftarrow \text{false}$; $V \leftarrow V \cup \{q\}$
- 4: **for** each pair $(q_a, q_b) \in V \times V$ **do**
- 5: $(q_a, q_b).res \leftarrow 1.0$; $(q_a, q_b).path \leftarrow \text{GENERATEPATH}(q_a, q_b)$; $E \leftarrow E \cup \{(q_a, q_b)\}$

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LAZYROADMAPCOLLISIONCHECKING

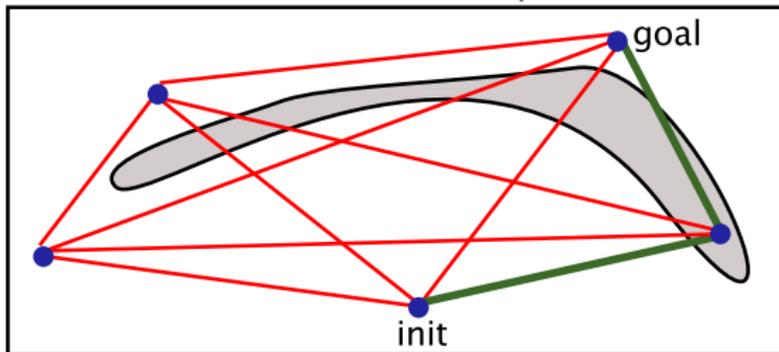
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2:    $[q_1, q_2, \dots, q_n] \leftarrow$  search  $G = (V, E)$  for sequence of edges connecting  $q_{init}$  to  $q_{goal}$ 
3:   for  $i = 1, 2, \dots, n$  do
4:     if  $q_i.checked = \text{false}$  and  $ISCONFIGCOLLISIONFREE(q_i) = \text{false}$  then
5:       remove  $q_i$  from roadmap; goto line 2
6:     else
7:        $q_i.checked \leftarrow \text{true}$ 
8:   while no edge collisions are found and minimum resolution not reached do
9:     for  $i = 1, 2, \dots, n - 1$  do
10:       $(q_i, q_{i+1}).res \leftarrow (q_i, q_{i+1}).res/2$ ; check  $(q_i, q_{i+1}).path$  at resolution  $(q_i, q_{i+1}).res$ 
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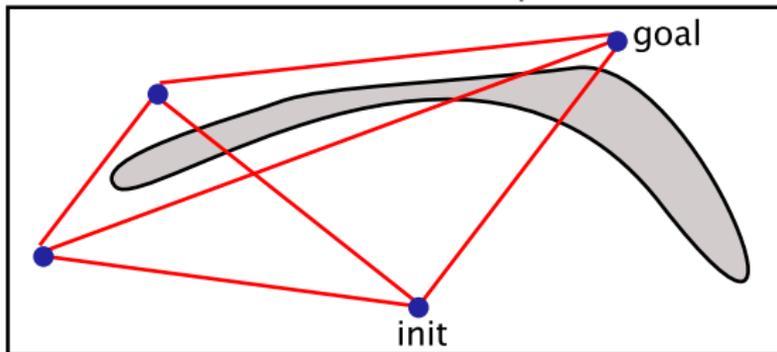
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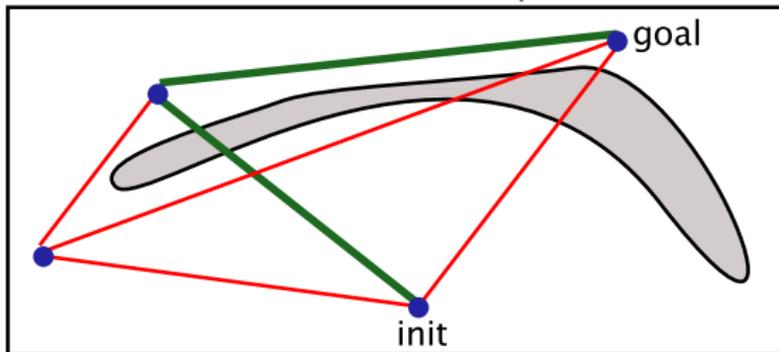
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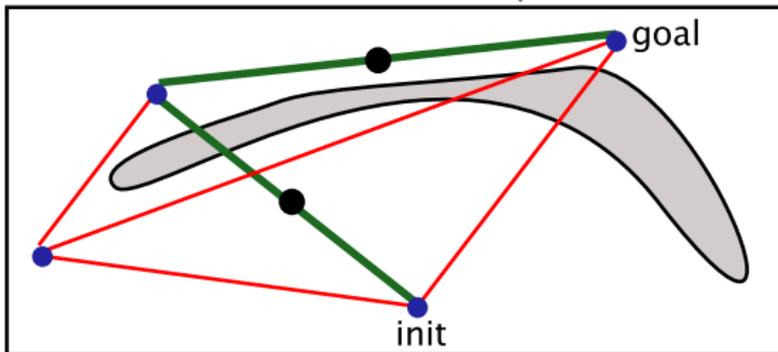
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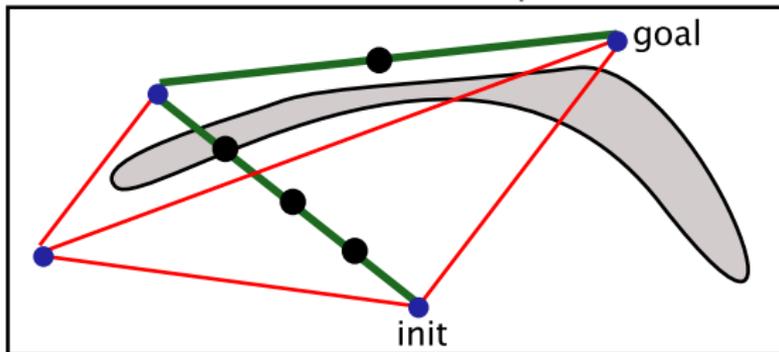
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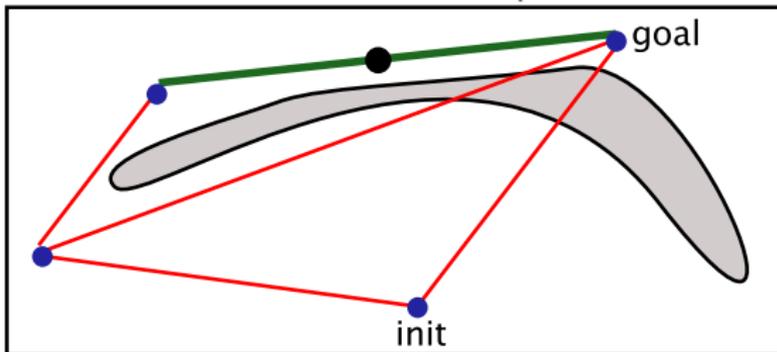
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```

Perform collision checking only when necessary

[Bohlin, Kavraki: Handbook on Randomized Computing 2000]



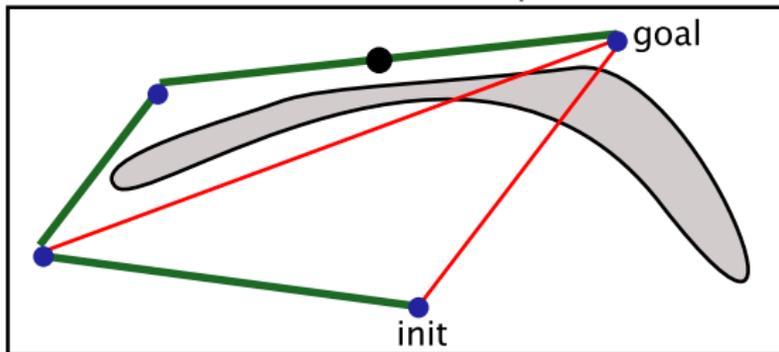
LAZYROADMAPCOLLISIONCHECKING

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LAZYROADMAPCOLLISIONCHECKING

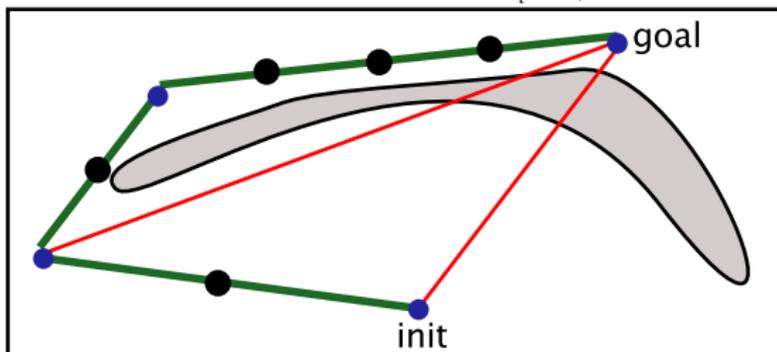
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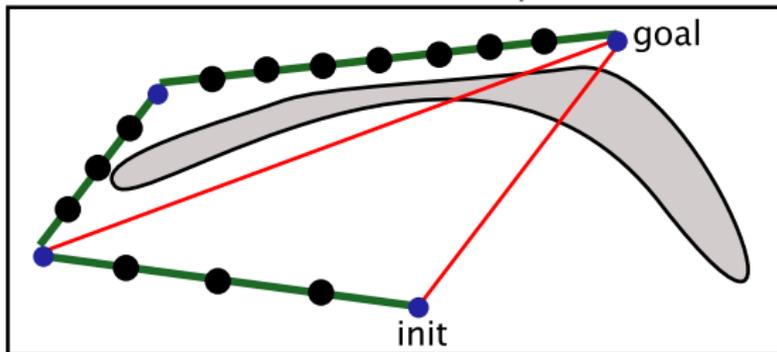
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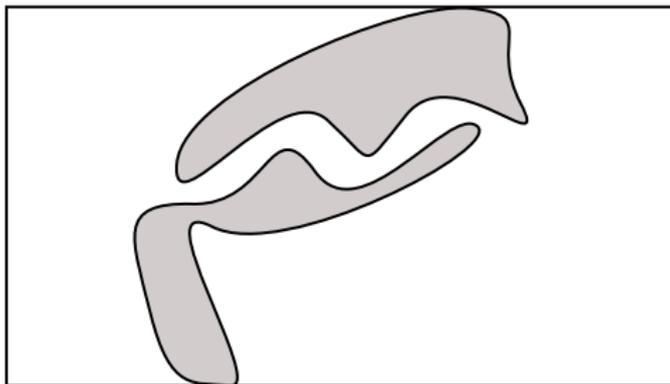
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```

Narrow-Passage Problem



- Probability of generating samples via uniform sampling in a narrow passage is low due to the small volume of the narrow passage
- Generating samples inside a narrow passage may be critical to the success of the path planner
- Objective is then to design sampling strategies that can increase the probability of generating samples inside narrow passages

Gaussian Sampling in PRM

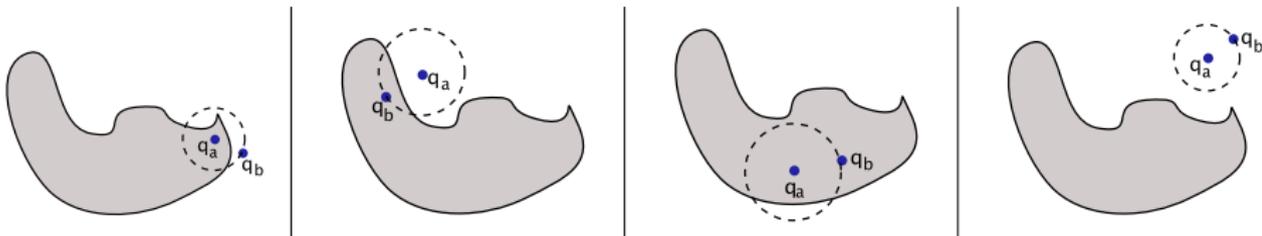
Objective: Increase Sampling Inside/Near Narrow Passages

Approach: Sample from a Gaussian distribution biased near the obstacles

GENERATECOLLISIONFREECONFIG

[Boor, Overmars, van Der Stappen: ICRA 1999]

- 1: $q_a \leftarrow$ generate config uniformly at random
- 2: $r \leftarrow$ generate distance from Gaussian distribution
- 3: $q_b \leftarrow$ generate config uniformly at random at distance r from q_a
- 4: $ok_a \leftarrow$ ISCONFIGCOLLISIONFREE(q_a)
- 5: $ok_b \leftarrow$ ISCONFIGCOLLISIONFREE(q_b)
- 6: **if** $ok_a = \text{true}$ **and** $ok_b = \text{false}$ **then return** q_a
- 7: **if** $ok_a = \text{false}$ **and $ok_b = \text{true}$ **then return** q_b**
- 8: **return** null



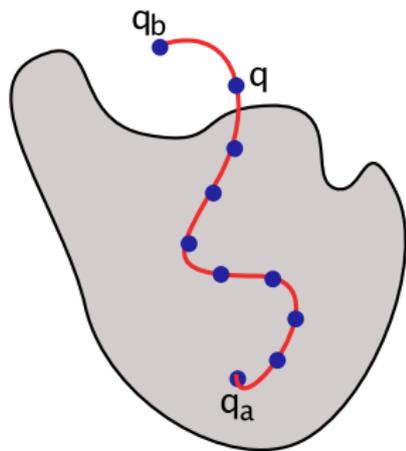
Obstacle-based Sampling in PRM

Objective: Increase Sampling Inside/Near Narrow Passages
Approach: Move samples in collision outside obstacle boundary

GENERATECOLLISIONFREECONFIG

[Amato, Bayazit, Dale, Jones, Vallejo: WAFR 1998]

```
1:  $q_a \leftarrow$  generate config uniformly at random
2: if ISCONFIGCOLLISIONFREE( $q_a$ ) = true then
3:   return  $q_a$ 
4: else
5:    $q_b \leftarrow$  generate config uniformly at random
6:   path  $\leftarrow$  GENERATEPATH( $q_a, q_b$ )
7:   for  $t = \delta$  to |path| by  $\delta$  do
8:     if ISCONFIGCOLLISIONFREE(path( $t$ )) then
9:       return path( $t$ )
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```



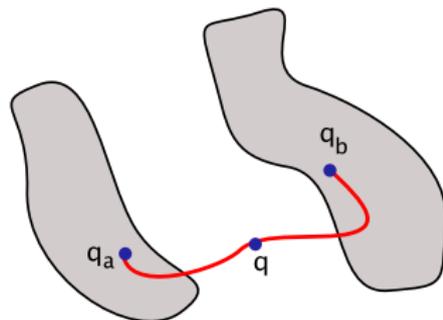
Bridge-based Sampling in PRM

Objective: Increase Sampling Inside/Near Narrow Passages
Approach: Create "bridge" between samples in collision

GENERATECOLLISIONFREECONFIG

- 1: $q_a \leftarrow$ generate config uniformly at random
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- 3: $ok_a \leftarrow$ ISCONFIGCOLLISIONFREE(q_a)
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- 5: **if** $ok_a = \text{false}$ **and** $ok_b = \text{false}$ **then**
- 6: $path \leftarrow$ GENERATEPATH(q_a, q_b)
- 7: $q \leftarrow path(0.5|path|)$
- 8: **if** ISCONFIGCOLLISIONFREE(q) **then**
- 9: **return** q
- 10: **return** null

[Hsu, Jiang, Reif, Sun: ICRA 2003]



Visibility-based Sampling in PRM

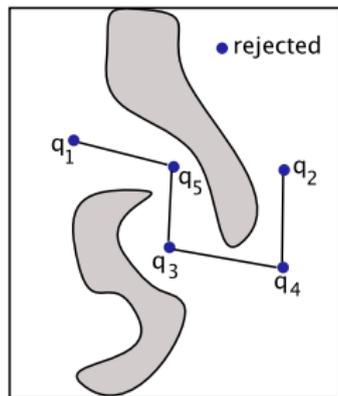
Objective: Capture connectivity of configuration space with few samples

Approach: Generate samples that create new components or join existing components

GENERATECOLLISIONFREECONFIG

[Nisseoux, Simeon, Laumond: Advanced Robotics J 2000]

- 1: $q \leftarrow$ generate config uniformly at random
- 2: **if** ISCONFIGCOLLISIONFREE(q) = true **then**
- 3: **if** q belongs to a new roadmap component **then**
- 4: **return** q
- 5: **if** q connects two roadmap components **then**
- 6: **return** q
- 7: **return** null



- q_1 : creates new roadmap component
- q_2 : creates new roadmap component
- q_3 : creates new roadmap component
- q_4 : connects two roadmap components
- q_5 : connects two roadmap components

Importance Sampling

Objective: Increase Sampling Inside/Near Narrow Passages

Approach: Improve roadmap connectivity

- Construct roadmap using given sampling strategy
- Identify roadmap nodes that lie in regions that are hard to connect
- Sample more in these regions

Importance Sampling

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- Select sample with probability $\frac{w(q)}{\sum_{q' \in V} w(q')}$
- Generate more samples around q
- Connect new samples to neighboring roadmap nodes

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- Each sampling strategy has its strengths and weakness
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- Sampler weight is updated based on quality of performance
- Balance between being “smart and slow” and “dumb and fast”

Proof Outline: Probabilistic Completeness of PRM

Components

- Free configuration space Q_{free} : arbitrary open subset of $[0, 1]^d$
- Local connector: connects $a, b \in Q_{\text{free}}$ via a straight-line path and succeeds if path lies entirely in Q_{free}
- Collection of roadmap samples from Q_{free}

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Let $a, b \in Q_{\text{free}}$ such that there exists a path γ between a and b lying in Q_{free} . Then the probability that PRM correctly answers the query (a, b) after generating n collision-free configurations is given by

$$\Pr[(a, b)\text{SUCCESS}] \geq 1 - \left[\frac{2L}{\sigma} \right] e^{\sigma \rho^d n},$$

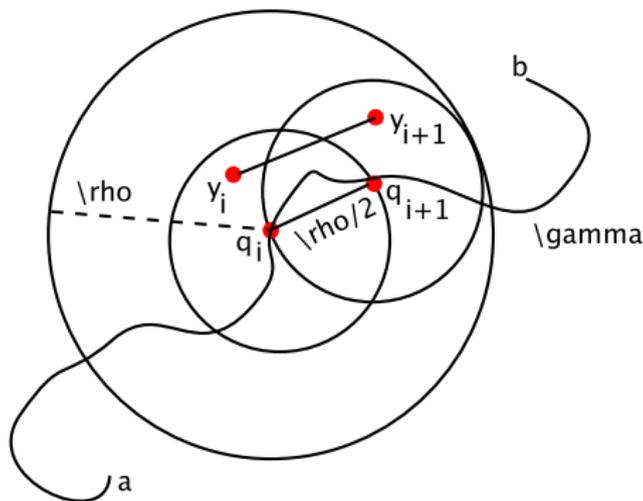
where

- L is the length of the path γ
- $\rho = \text{clr}(\gamma)$ is the clearance of path γ from obstacles
- $\sigma = \frac{\mu(B_1(\cdot))}{2^d \mu(Q_{\text{free}})}$
- $\mu(B_1(\cdot))$ is the volume of the unit ball in \mathbb{R}^d
- $\mu(Q_{\text{free}})$ is the volume of Q_{free}

Proof Outline: Probabilistic Completeness of PRM (cont.)

Basic Idea

- Reduce path to a set of open balls in Q_{free}
- Calculate probability of generating samples in those balls
- Connect samples in different balls via straight-line paths to compute solution path



Proof Outline: Probabilistic Completeness of PRM (cont.)

- Note that clearance $\rho = \text{clr}(\gamma) > 0$
- Let $m = \left\lceil \frac{2L}{\rho} \right\rceil$. Then, γ can be covered with m balls $B_{\rho/2}(q_i)$ where $a = q_1, \dots, q_m = b$
- Let $y_i \in B_{\rho/2}(q_i)$ and $y_{i+1} \in B_{\rho/2}(q_{i+1})$.
Then, the straight-line segment $\overline{y_i y_{i+1}} \in Q_{\text{free}}$, since $y_i, y_{i+1} \in B_{\rho}(q_i)$
- $I_i \stackrel{\text{def}}{=} \text{indicator variable that there exists } y \in V \text{ s.t. } y \in B_{\rho/2}(q_i)$
- $\Pr[(a, b)\text{FAILURE}] \leq \Pr[\bigvee_{i=1}^m I_i = 0] \leq \sum_{i=1}^m \Pr[I_i = 0]$
 - Note that $\Pr[I_i = 0] = \left(1 - \frac{\mu(B_{\rho/2}(q_i))}{\mu(Q_{\text{free}})}\right)^n$
i.e., probability that none of the n PRM samples falls in $B_{\rho/2}(q_i)$
 - I_i 's are independent because of uniform sampling in PRM

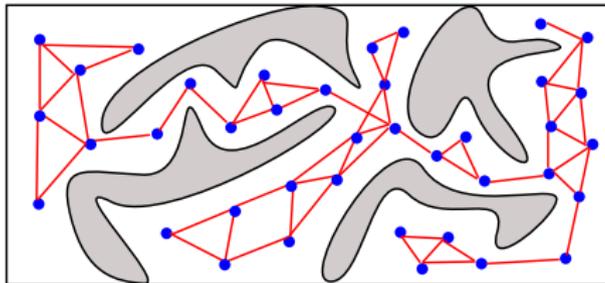
$$\text{Therefore, } \Pr[(a, b)\text{FAILURE}] \leq m \left(1 - \frac{\mu(B_{\rho/2}(\cdot))}{\mu(Q_{\text{free}})}\right)^n$$

$$\bullet \frac{\mu(B_{\rho/2}(\cdot))}{\mu(Q_{\text{free}})} = \left(\frac{\rho}{2}\right)^d \frac{\mu(B_1(\cdot))}{\mu(Q_{\text{free}})} = \sigma \rho^d$$

$$\text{Therefore, } \Pr[(a, b)\text{FAILURE}] \leq m (1 - \sigma \rho^d)^n \leq m e^{-\sigma \rho^d} = \left\lceil \frac{2L}{\rho} \right\rceil e^{-\sigma \rho^d} \quad \square$$

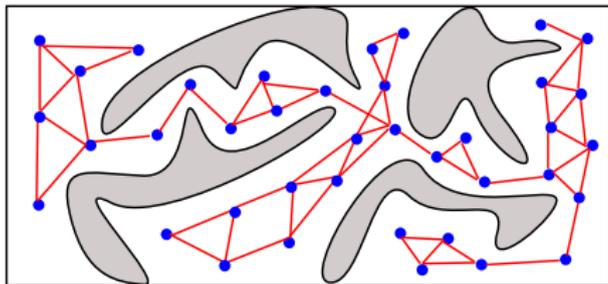
Motivation: Tree-based Motion Planning

- PRM-based planners aim to construct a roadmap that captures the whole connectivity of the configuration space



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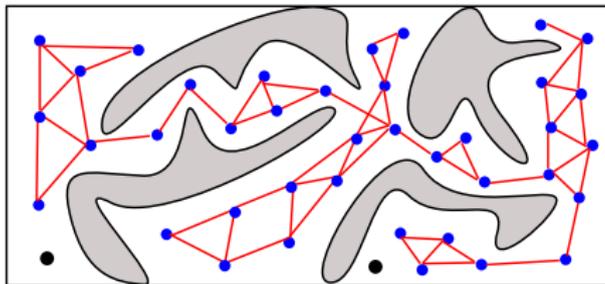
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- Good when the objective is to solve *multiple* queries

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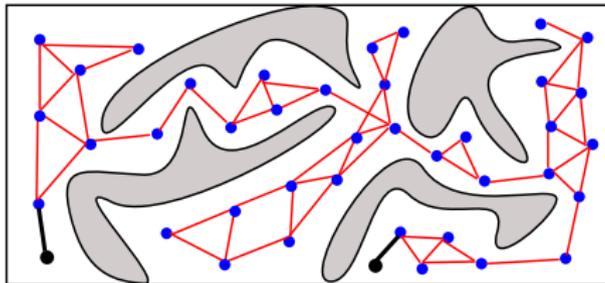
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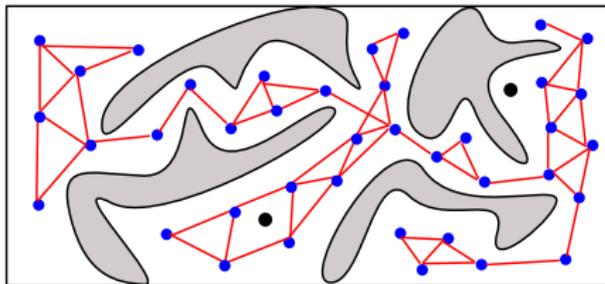
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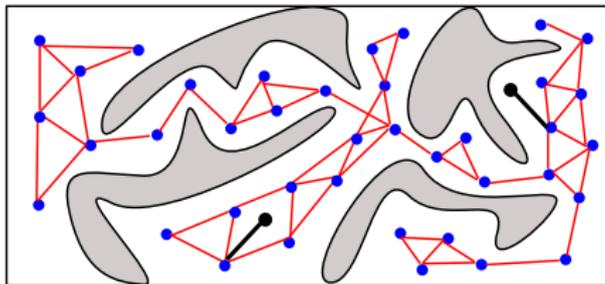
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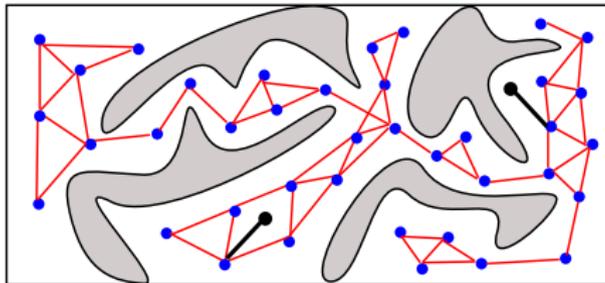
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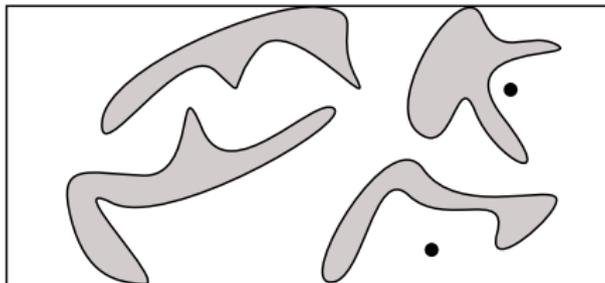
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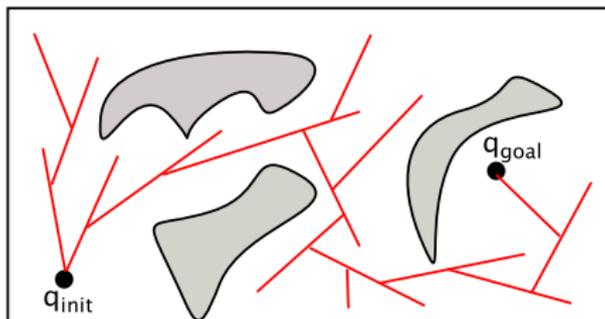


- Good when the objective is to solve *multiple* queries
- Maybe a bit too much when the objective is to solve a *single* query



General Idea

Grow a tree in the free configuration space from q_{init} toward q_{goal}



TREESearchFRAMEWORK(q_{init}, q_{goal})

1: $\mathcal{T} \leftarrow \text{ROOTTREE}(q_{init})$

2: **while** q_{goal} has not been reached **do**

3: $q \leftarrow \text{SELECTCONFIGFROMTREE}(\mathcal{T})$

4: $\text{ADDTREEBRANCHFROMCONFIG}(\mathcal{T}, q)$

Critical Issues

- How should a configuration be selected from the tree?
- How should a new branch be added to the tree from the selected configuration?

Rapidly-exploring Random Tree (RRT)

Pull the tree toward random samples in the configuration space

[LaValle, Kuffner: 1999]

- RRT relies on nearest neighbors and distance metric $\rho : Q \times Q \leftarrow \mathbb{R}^{\geq 0}$
- RRT adds Voronoi bias to tree growth

RRT($q_{\text{init}}, q_{\text{goal}}$)

▷ initialize tree

1: $\mathcal{T} \leftarrow$ create tree rooted at q_{init}

2: **while** solution not found **do**

▷ select configuration from tree

3: $q_{\text{rand}} \leftarrow$ generate a random sample

4: $q_{\text{near}} \leftarrow$ nearest configuration in \mathcal{T} to q_{rand} according to distance ρ

▷ add new branch to tree from selected configuration

5: path \leftarrow generate path (not necessarily collision free) from q_{near} to q_{rand}

6: **if** ISUBPATHCOLLISIONFREE(path, 0, step) **then**

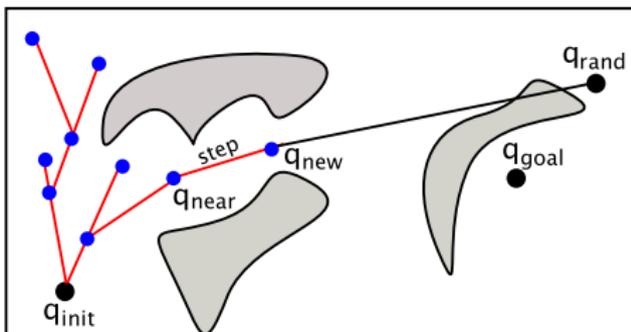
7: $q_{\text{new}} \leftarrow$ path(step)

8: add configuration q_{new} and edge ($q_{\text{near}}, q_{\text{new}}$) to \mathcal{T}

▷ check if a solution is found

9: **if** $\rho(q_{\text{new}}, q_{\text{goal}}) \approx 0$ **then**

10: **return** solution path from root to q_{new}



Rapidly-exploring Random Tree (RRT) (cont.)

Aspects for Improvement

Suggested Improvements in the Literature

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- BASICRRT does not take advantage of q_{goal}
- Tree is pulled towards random directions based on the uniform sampling of Q
- In particular, tree growth is not directed towards q_{goal}

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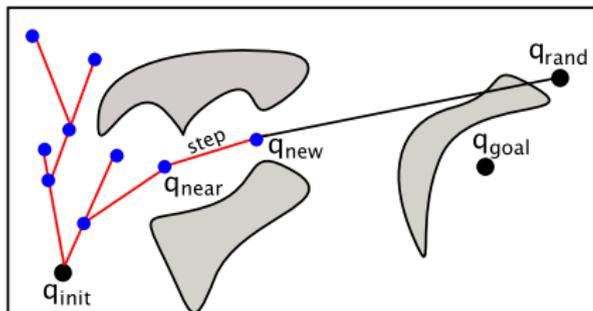
Suggested Improvements in the Literature

- Introduce goal-bias to tree growth (known as GOALBIASRRT)
 - q_{rand} is selected as q_{goal} with probability p
 - q_{rand} is selected based on uniform sampling of Q with probability $1 - p$
 - Probability p is commonly set to ≈ 0.05

Rapidly-exploring Random Tree (RRT) (cont.)

Aspects for Improvement

- BASICRRT takes only one small step when adding a new tree branch



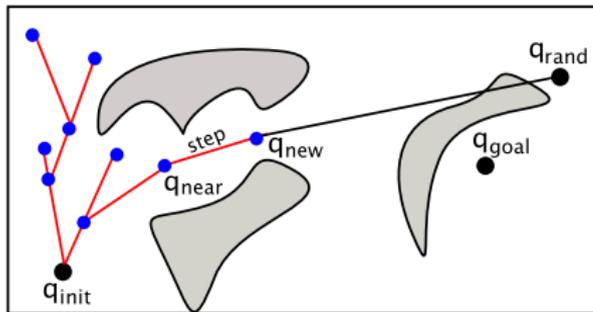
- This slows down tree growth

Suggested Improvements in the Literature

Rapidly-exploring Random Tree (RRT) (cont.)

Aspects for Improvement

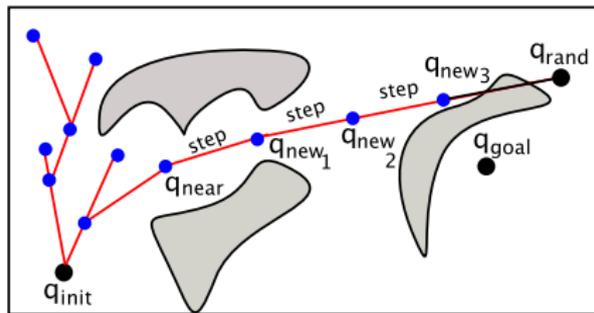
- BASICRRT takes only one small step when adding a new tree branch



- This slows down tree growth

Suggested Improvements in the Literature

- Take several steps until q_{rand} is reached or a collision is found (CONNECTRRT)
- Add all the intermediate nodes to the tree



Expansive-Space Tree (EST)

Push the tree frontier in the free configuration space

[Hsu, Rock, Motwani, Latombe: 1999]

Expansive-Space Tree (EST)

Push the tree frontier in the free configuration space

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- EST relies on a probability distribution to guide tree growth
- EST associates a weight $w(q)$ with each tree configuration q
- $w(q)$ is a running estimate on importance of selecting q as the tree configuration from which to add a new tree branch

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Push the tree frontier in the free configuration space

[Hsu, Rock, Motwani, Latombe: 1999]

- EST relies on a probability distribution to guide tree growth
- EST associates a weight $w(q)$ with each tree configuration q
- $w(q)$ is a running estimate on importance of selecting q as the tree configuration from which to add a new tree branch
 - $w(q) = \frac{1}{1+\text{deg}(q)}$
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[play movie]

Observations in High-Dimensional Problems

- Tree generally grows rapidly for the first few thousand iterations
- Tree growth afterwards slows down quite significantly
- Large number of configurations increases computational cost
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Possible improvements?

Bi-directional Trees

Grow two trees, rooted at q_{init} and q_{goal} , towards each other

- Bi-directional trees improve computational efficiency compared to a single tree
- Growth slows down significantly later than when using a single tree
- Fewer configurations in each tree, which imposes less of a computational burden
- Each tree explores a different part of the configuration space

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 from the two trees
 - 7: if successful, return path from q_{init} to q_{goal}
- Different tree planners can be used to grow each of the trees
 - E.g., RRT can be used for one tree and EST can be used for the other

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Sampling-based Roadmap of Trees (SRT)

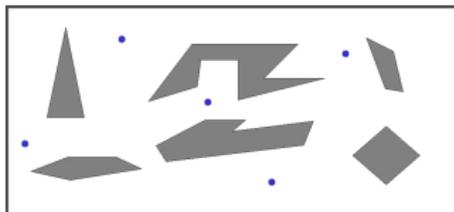
[Plaku, Bekris, Chen, Ladd, Kavraki: Trans on Robotics 2005]

- Hierarchical planner
- Top level performs global sampling (PRM-based)
- Bottom level performs local sampling (tree-based, e.g., RRT, EST)
- Combines advantages of global and local sampling

Sampling-based Roadmap of Trees (SRT) (cont.)

CREATETREESINROADMAP

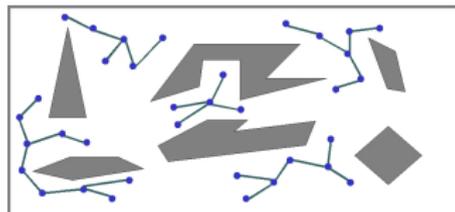
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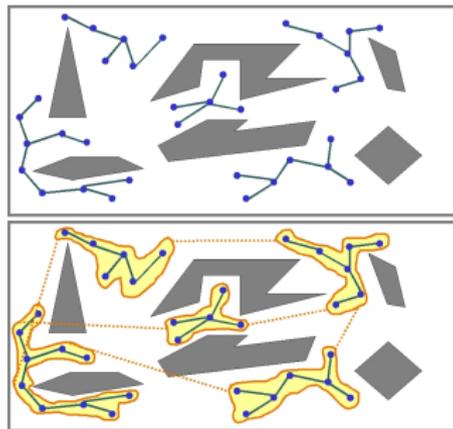
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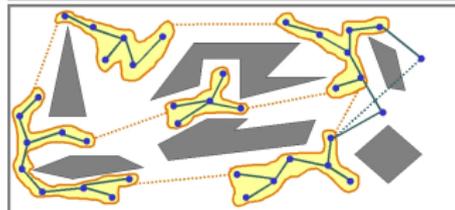
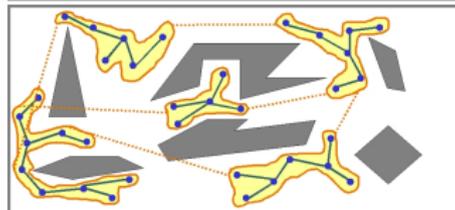
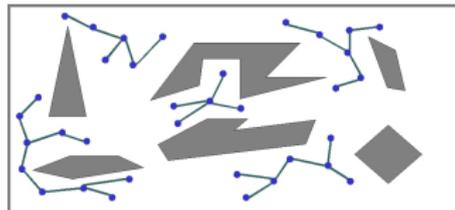
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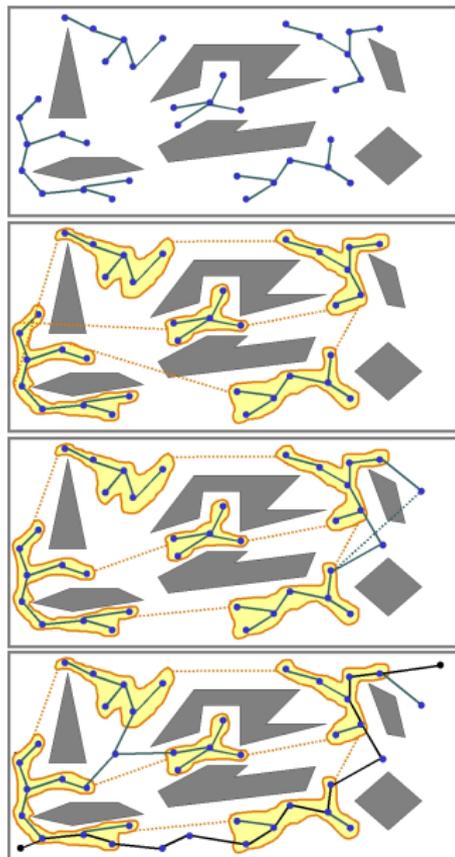
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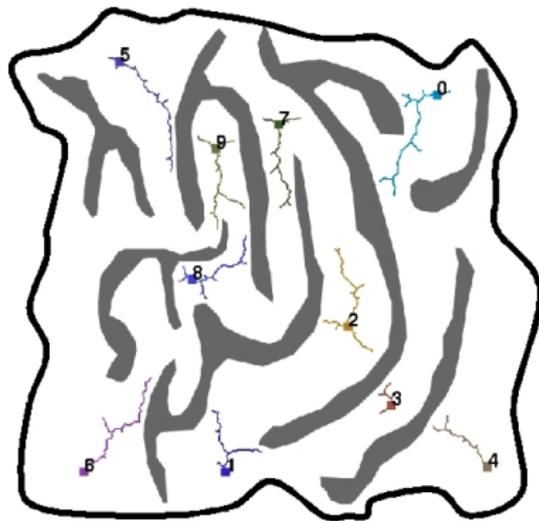
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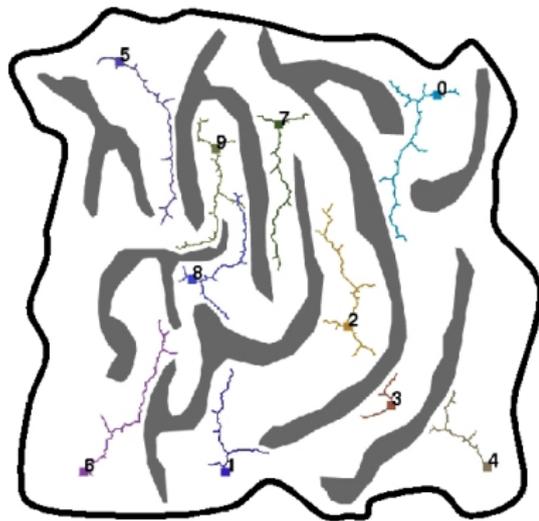
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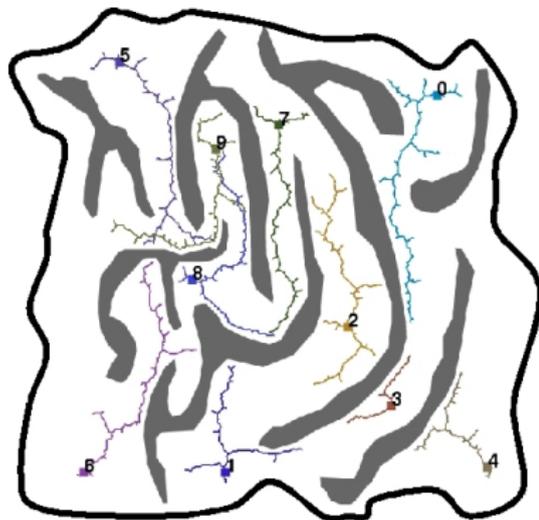
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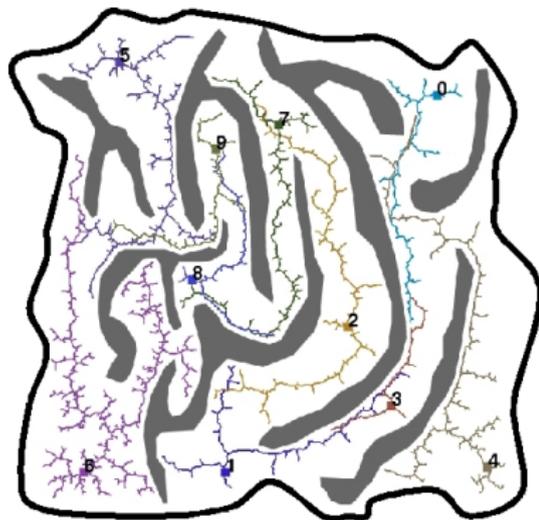
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Advantages

- Explores small subset of possibilities by sampling
- Computationally efficient
- Solves high-dimensional problems (with hundreds of DOFs)
- Easy to implement
- Applications in many different areas

Disadvantages

- Does not guarantee completeness (a complete planner always finds a solution if there exists one, or reports that no solution exists)

Is it then just a heuristic approach? No. It's more than that

It offers *probabilistic completeness*

- When a solution exists, a probabilistically complete planner finds a solution with probability as time goes to infinity.
- When a solution does not exist, a probabilistically complete planner may not be able to determine that a solution does not exist.