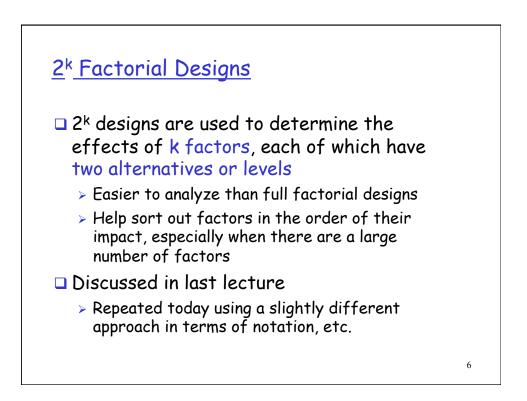
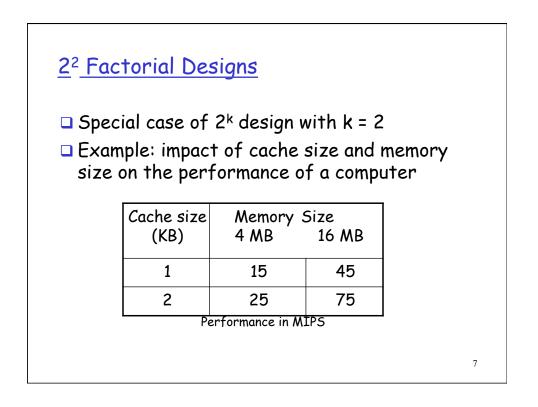
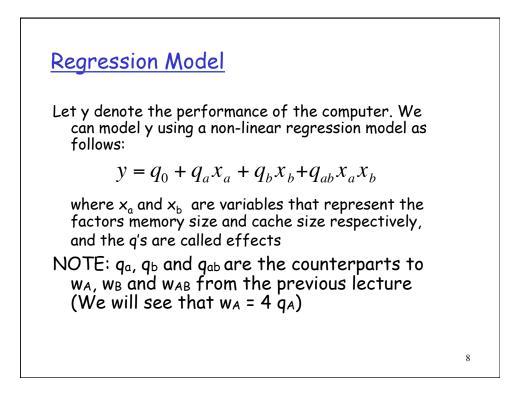


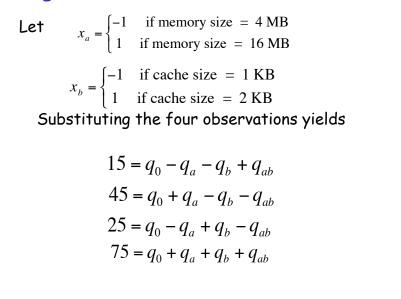
# 2<sup>k</sup> Factorial Designs

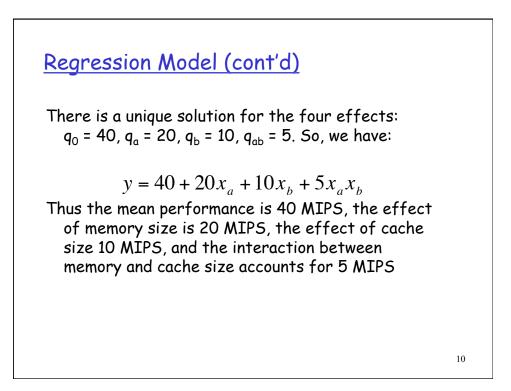






## Regression Model (con'td)





# Computing effects

In general, the model for a 2<sup>2</sup> design can be solved to obtain:

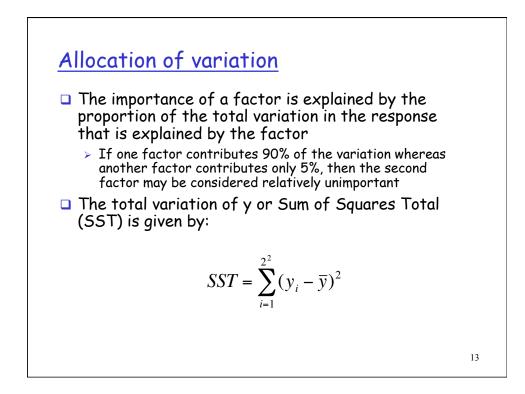
$$q_{0} = \frac{1}{4}(y_{1}+y_{2}+y_{3}+y_{4})$$

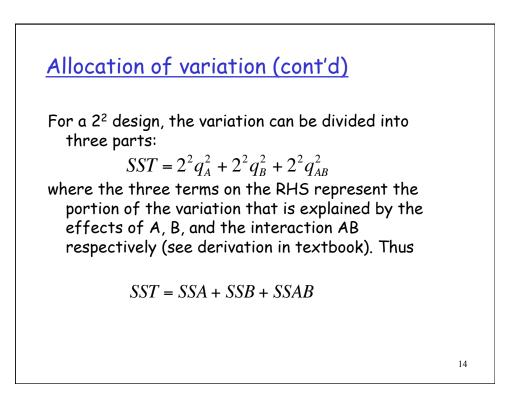
$$q_{A} = \frac{1}{4}(-y_{1}+y_{2}-y_{3}+y_{4})$$

$$q_{B} = \frac{1}{4}(-y_{1}-y_{2}+y_{3}+y_{4})$$

$$q_{AB} = \frac{1}{4}(y_{1}-y_{2}-y_{3}+y_{4})$$

I	A	В	AB	У
1	-1	-1	1	15
1	1	-1	-1	45
1	-1	1	-1	25
1	1	1	1	75
160	80	40	20	Total
40	20	10	5	Total/4



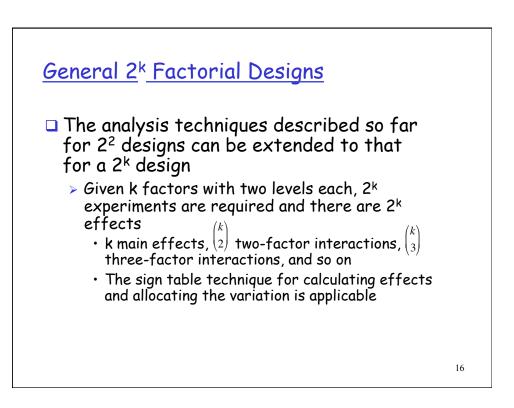


#### Example

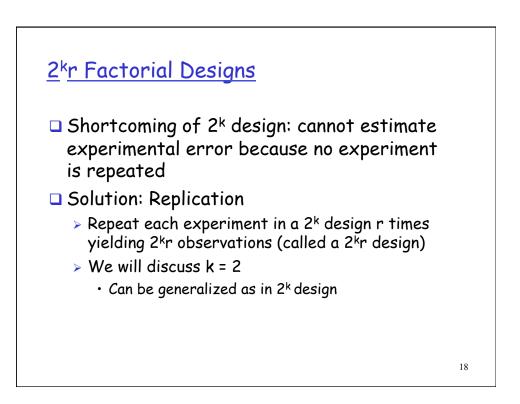
For the memory-cache example,

$$\overline{y} = \frac{1}{4}(15 + 55 + 25 + 75) = 40$$
  
Total variation 
$$= \sum_{i=1}^{4} (y_i - \overline{y})^2 = (25^2 + 15^2 + 15^2 + 35^2)$$
$$= 2100 = 4 \times 20^2 + 4 \times 10^2 + 4 \times 5^2$$

Thus, 76% (1600) of the total variation can be attributed to memory size, 19% (400) can be attributed to cache, and only 5% (100) can be attributed to the interaction between memory and cache.









Model for 2<sup>2</sup> design is extended to add an error term

 $y = q_0 + q_A x_A + q_B x_B + q_{AB} x_A x_B + e$ 

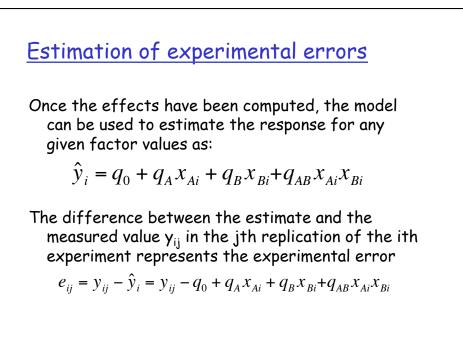
- where the q's are effects as before and e is the experimental error
- Holding the factor level constant and repeating the experiment yields samples of the response y<sub>i</sub>
- Statistical analysis of the y<sub>i</sub>'s yields the fraction of variation due to experimental error, and confidence intervals for y

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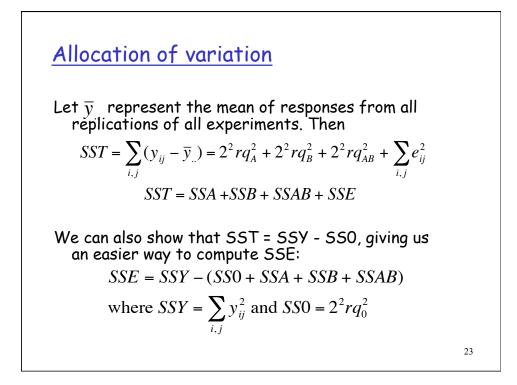
### **Computation of Effects**

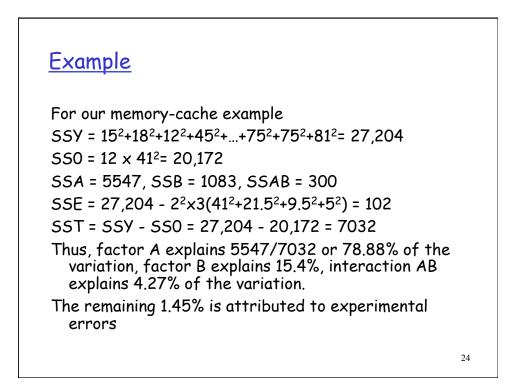
The effects can be calculated using a sign table as before except that in the y column we put the sample mean of r measurements at the given factor level

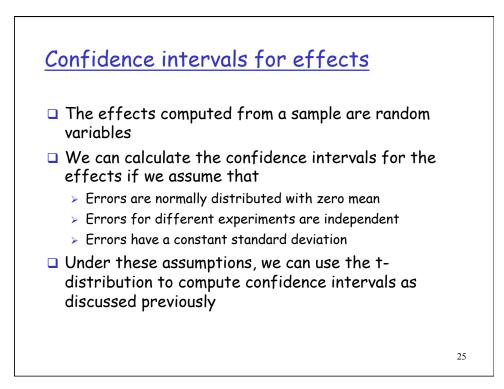
I	Α	В	AB	У	Mean $\overline{y}$
1	-1	-1	1	(15,18,12)	15
1	1	-1	-1	(45,48,51)	48
1	-1	1	-1	(25,28,19)	24
1	1	1	1	(75,75,81)	77
164	86	38	20		Total
41	21.5	9.5	5		Total/4

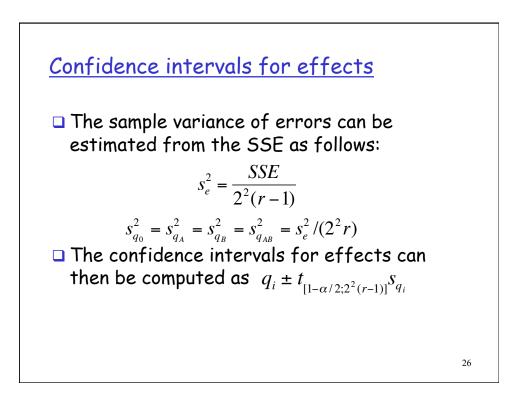


I	A	В	AB	(y <sub>i1</sub> ,y <sub>i2</sub> ,y <sub>i3</sub> )	$\hat{y}_i$	e <sub>i1</sub>	e <sub>i2</sub>	e <sub>i3</sub>
1	-1	-1	1	(15,18,12)	15	0	3	-3
1	1	-1	-1	(45,48,51)	48	-3	0	3
1	-1	1	-1	(25,28,19)	24	1	4	-5
1	1	1	1	(75,75,81)	77	-2	-2	4
41	21.5	9.5	5					





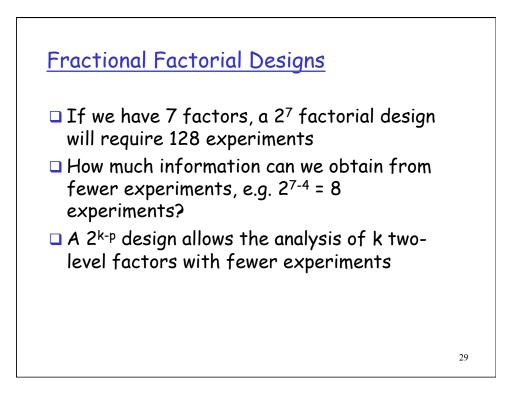




# Example • For the memory-cache example, $s_e = \sqrt{\frac{SSE}{2^2(r-1)}} = \sqrt{\frac{102}{8}} = \sqrt{12.75} = 3.57$ $s_{q_i} = s_e / \sqrt{(2^2 r)} = 3.57 / \sqrt{12} = 1.03$ • The t-value for 8 degrees of freedom and 90% confidence is 1.86. Thus, the confidence intervals for the effects are $q_i \pm (1.86)(1.03) = q_i \pm 1.92$ that is (39.08,42.91), (19.58,23.41), (7.58,11.41), (3.08,6.91) for $q_0$ , $q_A$ , $q_B$ , and $q_{AB}$ respectively

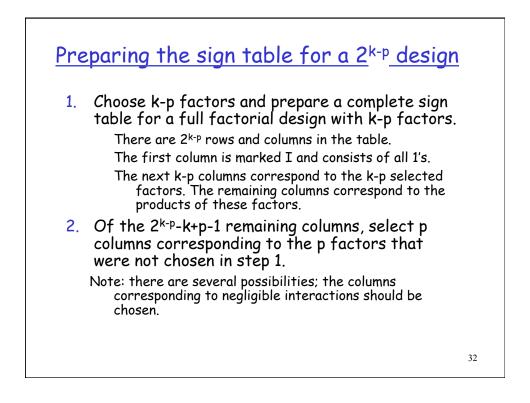
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Consider the 2 <sup>3</sup>	de	sigr	1 be	low:					
Experiment #	I	A	В	С	AB	AC	BC	ABC	
1	1	-1	-1	-1	1	1	1	-1	
2	1	1	-1	-1	-1	-1	1	1	
3	1	-1	1	-1	-1	1	-1	1	
4	1	1	1	-1	1	-1	-1	-1	
5	1	-1	-1	1	1	-1	-1	1	
6	1	1	-1	1	-1	1	-1	-1	
7	1	-1	1	1	-1	-1	1	-1	
8	1	1	1	1	1	1	1	1	

A	2 <sup>7-4</sup> desi	gn								
If the in	teractions AB, /	AC, AD,	, ABCI	) are n	egligible	e we c	an use	the <sup>.</sup>	table b	below
	Experiment #	I	Α	В	С	D	E	F	G	У
	1	1	-1	-1	-1	1	1	1	-1	20
	2	1	1	-1	-1	-1	-1	1	1	35
	3	1	-1	1	-1	-1	1	-1	1	7
	4	1	1	1	-1	1	-1	-1	-1	42
	5	1	-1	-1	1	1	-1	-1	1	36
	6	1	1	-1	1	-1	1	-1	-1	50
	7	1	-1	1	1	-1	-1	1	-1	45
	8	1	1	1	1	1	1	1	1	82
	Total	317	101	35	109	43	1	47	3	
	Total/8	39.62 <mark>(</mark>	12.62	4.37	13.62	5.37	0.12	5.9	0.37	
	Percent variation		37.26	4.74	43.4	6.75	0	8.1	0.03	31
										31



## <u>A 2<sup>4-1</sup> design</u>

Experiment #	I	A	В	С	AB	AC	BC	ABC
1	1	-1	-1	-1	1	1	1	-1
2	1	1	-1	-1	-1	-1	1	1
3	1	-1	1	-1	-1	1	-1	1
4	1	1	1	-1	1	-1	-1	-1
5	1	-1	-1	1	1	-1	-1	1
6	1	1	-1	1	-1	1	-1	-1
7	1	-1	1	1	-1	-1	1	-1
8	1	1	1	1	1	1	1	1

If the ABC interaction is negligible, we should replace ABC with D. If AB is negligible, we can replace AB with D.

