Relational Algebra 1

Week 4

Relational Query Languages

- *Query languages:* Allow manipulation and retrieval of data from a database.
- Relational model supports simple, powerful QLs:
  - Strong formal foundation based on logic.
  - Allows for much optimization.
- Query Languages **!=** programming languages!
  - QLs not expected to be "Turing complete".
  - QLs not intended to be used for complex calculations.
  - QLs support easy, efficient access to large data sets.

Formal Relational Query Languages

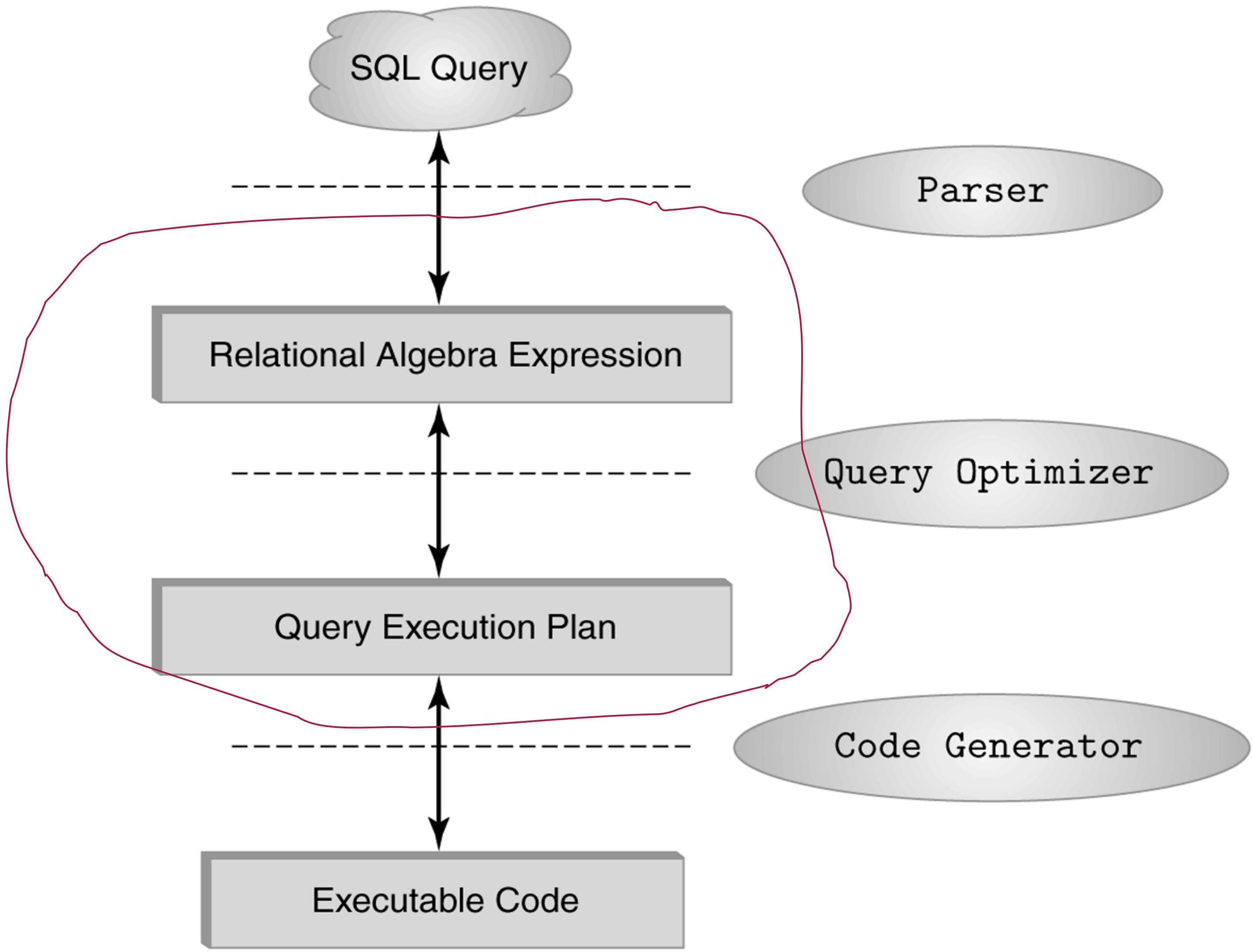
Two mathematical Query Languages form the basis for “real” languages (e.g. SQL), and for implementation:

❶ *Relational Algebra*: More operational, very useful for representing execution plans.

❷ *Relational Calculus*: Lets users describe what they want, rather than how to compute it. (Non-operational, *declarative*.)

☛ *Understanding Algebra is key to understanding SQL, and query processing!*

The Role of Relational Algebra in a DBMS



Algebra Preliminaries

- A query is applied to *relation instances*, and the result of a query is also a relation instance.
  - *Schemas* of input relations for a query are fixed (but query will run regardless of instance!)
  - The schema for the *result* of a given query is also fixed! Determined by definition of query language constructs.

Relational Algebra

- Procedural language
- Five basic operators

SQL is closely based on relational algebra.

• **selection**	select
• **projection**	project
• **union**	(why no intersection?)
• **set difference**	difference
• **Cross product**	Cartesian product

- The are some other operators which are composed of the above operators. These show up so often that we give them special names.
- The operators take one or two relations as inputs and give a new relation as a result.

Select Operation – Example

- Relation *r*

A	B	C	D
α	α	1	7
α	β	5	7
β	β	12	3
β	β	23	10

- $\sigma_{A=B \wedge D > 5}(r)$

A	B	C	D
α	α	1	7
β	β	23	10

lowercase
Greek sigma

Intuition: The **select** operation allows us to retrieve some rows of a relation (by "some" I mean anywhere from none of them to all of them)

Here I have retrieved all the rows of the relation *r* where the value in field *A* equals the value in field *B*, and the value in field *D* is greater than 5.

Select Operation

- Notation: $\sigma_p(r)$ lowercase Greek sigma σ
- p is called the **selection** predicate
- Defined as:

$$\sigma_p(\mathbf{r}) = \{t \mid t \in r \textbf{ and } p(t)\}$$

Where p is a formula in propositional calculus consisting of terms connected by : ∧ (**and**), ∨ (**or**), ¬ (**not**)
Each term is one of:

<attribute> *op* <attribute> or <constant>

where *op* is one of: =, ≠, >, ≥, <, ≤

- Example of selection:

$\sigma_{name = \text{'Lee'}}(professor)$

Project Operation – Example I

- Relation *r*:

A	B	C
α	10	7
α	20	1
β	30	1
β	40	2

<u>Intuition</u>: The **project** operation allows us to retrieve some columns of a relation (by "some" I mean anywhere from none of them to all of them)

- $\pi_{A,C}(r)$

A	C
α	7
α	1
β	1
β	2

Here I have retrieved columns *A* and *C*.

Greek lower-case pi

Project Operation – Example II

- Relation r:

A	B	C
α	10	1
α	20	1
β	30	1
β	40	2

- $\pi_{A,C}(r)$

A	C
α	1
α	1
β	1
β	2

=

A	C
α	1
β	1
β	2

<u>Intuition</u>: The project operation removes duplicate rows, since relations are sets.

Here there are two rows with $A = \alpha$ and $C = 1$. So one was discarded.

Project Operation

- Notation:

$$\pi_{A1,\ A2,\ \ldots,\ Ak}\ (r)$$

Greek lower-case pi

where A_1, A_2 are attribute names and r is a relation name.

- The result is defined as the relation of k columns obtained by erasing the columns that are not listed
- Duplicate rows removed from result, since relations are sets.

Union Operation – Example

Relations *r, s:*

r

A	B
α	1
α	2
β	1

s

A	B
α	2
β	3

r ∪ s:

A	B
α	1
α	2
β	1
β	3

Intuition: The **union** operation concatenates two relations, and removes duplicate rows (since relations are sets).

Here there are two rows with $A = \alpha$ and $B = 2$. So one was discarded.

Union Operation

- Notation: $r \cup s$
- Defined as:

$$r \cup s = \{t \mid t \in r \text{ or } t \in s\}$$

"Union-compatible"

For $r \cup s$ to be valid.

1. r, s must have the *same arity* (same number of attributes)
2. The attribute domains must be *compatible* (e.g., 2nd column of r deals with the same type of values as does the 2nd column of s).

Although the field types must be the same, the names can be different. For example I can union *professor* and *lecturer* where:

professor(<u>*PID*</u> : string, *name* : string)
lecturer(<u>*LID*</u> : string, *first_name* : string)

Related Operation: Intersection

Relations *r, s:*

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

$r \cap s$:

A	B
α	2

- Similar to Union operation.
- But Intersection is NOT one of the five basic operations.
- Intuition: The **intersection** operation computes the common rows between two relations

Set Difference Operation – Example

Relations *r, s:*

r

A	B
α	1
α	2
β	1

s

A	B
α	2
β	3

r – *s*:

A	B
α	1
β	1

Intuition: The **set difference** operation returns all the rows that are in *r* but not in *s*.

Set Difference Operation

- Notation $r - s$
- Defined as:

$$r - s = \{t \mid t \in r \textbf{ and } \mathrm{t} \notin s\}$$

- Set differences must be taken between *compatible* relations.
  "Union-compatible"
  - r and s must have the *same arity*
  - attribute domains of r and s must be compatible
- Note that in general $r - s \neq s - r$

Cross-Product Operation-Example

Relations *r, s*:

r

A	B
α	1
β	2

s

C	D	E
α	10	a
β	10	a
β	20	b
γ	10	b

r x *s*:

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

Intuition: The **cross product** operation returns all possible combinations of rows in *r* with rows in *s*.

In other words the result is every possible pairing of the rows of *r* and *s*.

Cross-Product Operation

- Notation $r \times s$
- Defined as:

  $$r \times s = \{t\, q \mid t \in r \textbf{ and } q \in s\}$$

- Assume that attributes of r(R) and s(S) are disjoint. (That is, $R \cap S = \emptyset$).
- If attributes names of *r(R)* and *s(S)* are not disjoint, then renaming must be used.

Composition of Operations

- We can build expressions using multiple operations
- Example: $\sigma_{A=C}(r \times s)$

A	B
α	1
β	2

r

C	D	E
α	10	a
β	10	a
β	20	b
γ	10	b

s

r x *s*:

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

$\sigma_{A=C}(r \times s)$ ⇨

A	B	C	D	E
α	1	α	10	a
β	2	β	10	a
β	2	β	20	b

"take the cross product of *r* and *s*, then return only the rows where *A* equals *B*"

Rename Operation

- Allows us to name, and therefore to refer to, the results of relational-algebra expressions.

Example:

$\rho\ (myRelation,\ (r - s))$

Renaming columns (rename A to A2):

$\rho\ (myRelation(A\text{->}A2),\ (r - s))$

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

Take the set difference of r and s, and call the result myRelation
Renaming in relational algebra is essentiality the same as assignment in a programming language

A	B
α	1
β	1

myRelation

Rename Operation

If a relational-algebra expression Y has arity n, then

$$\rho(X(A\text{->}A1,\ B\text{->}A2,\ \ldots),\ Y)$$

returns the result of expression Y under the name X, and with the attributes renamed to $A1, A2, \ldots., An$.

For example,

$\rho\ (myRelation(A\text{->}E,\ B\text{->}K),\ (r - s))$

Take the set difference of r and s, and call the result myRelation, while renaming the first field to E, and the second field to *K*.

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

E	K
α	1
β	1

myRelation

Sailors Example

Sailors(sid, sname, rating, age)

Boats(bid, bname, color)

Reserves(sid, bid, day)

Example Instances

- "Sailors" and "Reserves" relations for our examples.

R1

sid	bid	day
22	101	10/10/96
58	103	11/12/96

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Algebra Operations

- Look what we want to get from the following table:

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Selection

- Selects rows that satisfy *selection condition*.
- No duplicates in result! (Why?)
- *Schema* of result identical to schema of (only) input relation.

$S2$

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

$$\sigma_{rating > 8}(S2) =$$

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

Projection

- Deletes attributes that are not in *projection list*.
- *Schema* of result contains exactly the fields in the projection list, with the same names that they had in the (only) input relation.
- Projection operator has to eliminate *duplicates*! (Why??)
  - Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it.

sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

$$\pi_{sname,rating}(S2)$$

age
35.0
55.5

$$\pi_{age}(S2)$$

Composition of Operations

- *Result* relation can be the *input* for another relational algebra operation! (*Operator composition*)

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

$$\pi_{sname,rating}(\sigma_{rating>8}(S2)) =$$

sname	rating
yuppy	9
rusty	10

What do we want to get from two relations?

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

R1

sid	bid	day
22	101	10/10/96
58	103	11/12/96

What about: Who reserved boat 101?

Or: Find the name of the sailor who reserved boat 101.

Cross-Product

- Each row of S1 is paired with each row of R1.
- *Result schema* has one field per field of S1 and R1, with field names inherited.

sid1	sname	rating	age	sid2	bid	day
22	dustin	7	45.0	22	101	10/10/96
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	22	101	10/10/96
31	lubber	8	55.5	58	103	11/12/96
58	rusty	10	35.0	22	101	10/10/96
58	rusty	10	35.0	58	103	11/12/96

☛ *Renaming operator (because of naming conflict)*:

$$\rho(sid \rightarrow sid1,\ S1) \times \rho(sid \rightarrow sid2,\ R1)$$

Why does this cross product help

Query: Find the name of the sailor who reserved boat 101.

Another example

- Find the name of the sailor having the highest rating.

$$\text{AllR} = \pi_{ratingA}\,\rho(rating \text{->} ratingA,\ S2)$$

$$\text{Result?} = \pi_{Sname}(\sigma_{rating < ratingA}(S2 \times \text{AllR}))$$

What's in "Result?" ?

Does it answer our query?

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

×

AllR

ratingA
9
8
5
10

=

sid	sname	rating	age	ratingA
28	yuppy	9	35.0	9
28	yuppy	9	35.0	8
28	yuppy	9	35.0	5
28	**yuppy**	**9**	**35.0**	**10**
31	**lubber**	**8**	**55.5**	**9**
31	lubber	8	55.5	8
31	lubber	8	55.5	5
31	**lubber**	**8**	**55.5**	**10**
44	**guppy**	**5**	**35.0**	**9**
44	**guppy**	**5**	**35.0**	**8**
44	guppy	5	35.0	5
44	**guppy**	**5**	**35.0**	**10**
58	rusty	10	35.0	9
58	rusty	10	35.0	8
58	rusty	10	35.0	5
58	rusty	10	35.0	10

$$\text{AllR} = \pi_{ratingA}\, \rho(rating \text{->} ratingA, S2)$$

$$\text{Result?} = \pi_{Sname}(\sigma_{rating<ratingA}(S2 \times \text{AllR}))$$

Union, Intersection, Set-Difference

- All of these operations take two input relations, which must be *union-compatible*:
  - Same number of fields.
  - 'Corresponding' fields have the same type.
- What is the *schema* of result?

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0
44	guppy	5	35.0
28	yuppy	9	35.0

$S1 \cup S2$

sid	sname	rating	age
22	dustin	7	45.0

$S1 - S2$

sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

$S1 \cap S2$

Back to our query

- Find the name of the sailor having the highest rating.

Relational Algebra (Summary)

- Basic operations:
  - *Selection* (σ) Selects a subset of rows from relation.
  - *Projection* (π) Deletes unwanted columns from relation.
  - *Cross-product* ($\times$) Allows us to combine two relations.
  - *Set-difference* (-) Tuples in reln. 1, but not in reln. 2.
  - *Union* ($\cup$) Tuples in reln. 1 and in reln. 2.

  Also,
  - *Rename* (ρ) Changes names of the attributes
- Since each operation returns a relation, operations can be *composed*! (Algebra is "closed".)
- Use of temporary relations recommended.