Schema Refinement & Normalization Theory

Functional Dependencies

Week 14

Decomposition into BCNF

- Recall that for X → A in *F* over R to satisfy BCNF requirement, one of the followings must be true:
  - XA are not all in R, <u>or</u>
  - X → A is trivial, i.e. A is in X, <u>or</u>
  - X is a superkey, i.e. X → R is in F^+

- Consider relation R with FDs F. If X → A in *F* over R violates BCNF, i.e.,
  - XA are all in R, <u>and</u>
  - A is not in X, <u>and</u> → non-trivial FD
  - X → R is not in F^+ → X is not a superkey

Decomposition into BCNF

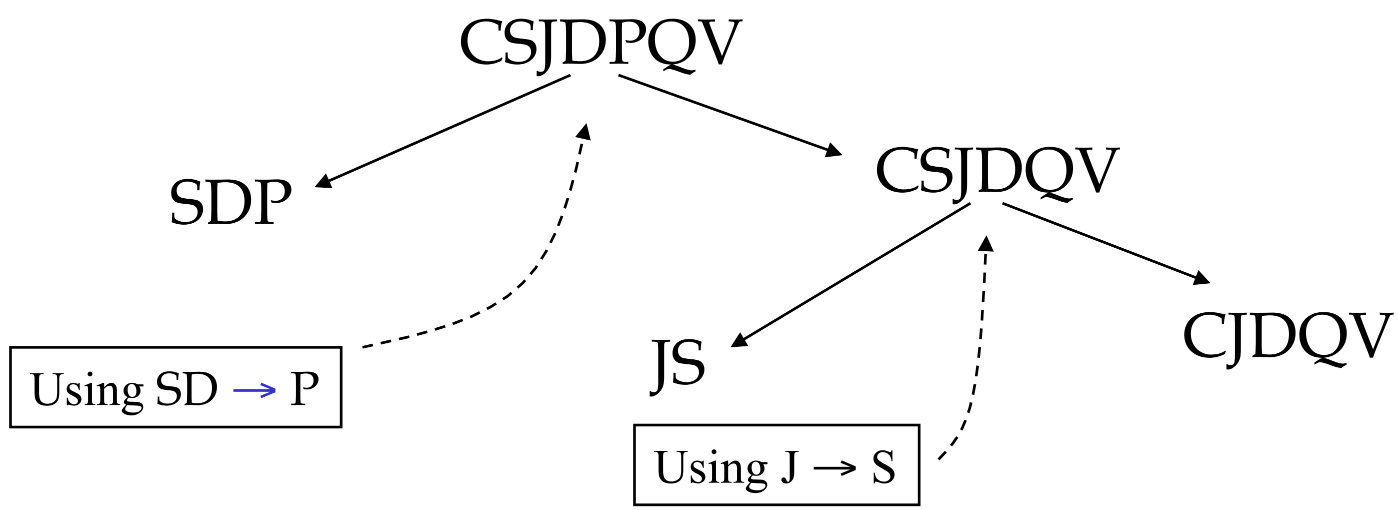
- Consider relation R with FDs F. If $X \rightarrow A$ in F over R violates BCNF, i.e.,
  - XA are all in R, and
  - A is not in X, and → non-trivial FD
  - $X \rightarrow R$ is not in F^+ → X is not a (super)key
- Then: decompose R into R - A and XA.
- Repeated application of this idea will give us a collection of relations that are in BCNF; lossless join decomposition, and guaranteed to terminate.

BCNF Decomposition Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B; B \rightarrow C\}$
  Key = $\{A\}$
- R is not in BCNF ($B \rightarrow C$ but B is not a superkey)
- Decomposition
  - $R_1 = (B, C)$
  - $R_2 = (A, B)$

BCNF Decomposition Example 2

- Assume relation schema CSJDPQV:
  Contracts(*contract_id, supplier, project, dept, part, qty, value*)
  - key C, JP → C, SD → P, J → S
- To deal with SD → P, decompose into SDP, CSJDQV.
- To deal with J → S, decompose CSJDQV into JS and CJDQV
- A tree representation of the decomposition:



BCNF Decomposition

- In general, several dependencies may cause violation of BCNF. The order in which we “deal with” them could lead to very different sets of relations!

How do we know R is in BCNF?

- If R has only two attributes, then it is in BCNF
- If F only uses attributes in R, then:
  - R is in BCNF *if and only if* for each $X \rightarrow Y$ in F (***not*** F^+***!***), X is a superkey of R, i.e., $X \rightarrow R$ is in F^+ (not F!).

Checking for BCNF Violations

- List all non-trivial FDs
- Ensure that left hand side of each FD is a superkey
- We have to first find all the keys!

Checking for BCNF Violations

- Is Courses(course_num, dept_name, course_name, classroom, enrollment, student_name, address) in BCNF?
- FDs are:
  - course_num, dept_name → course_name
  - course_num, dept_name → classroom
  - course_num, dept_name → enrollment
- What is (course_num, dept_name)$^+$?
  - {course_num, dept_name, course_name, classroom, enrollment}
- Therefore, the key is

  {course_num, dept_name, course_name, classroom, enrollment, student_name, address}
- The relation is not in BCNF

BCNF and Dependency Preservation

- In general, there may not be a dependency preserving decomposition into BCNF.
- Example: schema CSZ (city, street_name, zip_code) with FDs: CS $\rightarrow$ Z, Z $\rightarrow$ C

  $$(city, street\_name) \rightarrow zip\_code$$

  $$zip\_code \rightarrow city$$

- Can't decompose while ***preserving*** CS $\rightarrow$ Z, but CSZ is not in BCNF.

Example Regarding Dependency Preservation

- $R = (A, B, C)$
  $F = \{A \rightarrow B, B \rightarrow C)$
  - Can be decomposed in two different ways
- $R_1 = (A, B), \quad R_2 = (B, C)$
  - Lossless-join decomposition:

    $$R_1 \cap R_2 = \{B\} \text{ and } B \rightarrow BC$$
  - Dependency preserving
- $R_1 = (A, B), \quad R_2 = (A, C)$
  - Lossless-join decomposition:

    $$R_1 \cap R_2 = \{A\} \text{ and } A \rightarrow AB$$
  - Not dependency preserving
    (cannot check $B \rightarrow C$ without computing $R_1 \bowtie R_2$)

Dependency Preserving Decomposition

- Consider CSJDPQV, C is key, JP $\rightarrow$ C and SD $\rightarrow$ P.
  - BCNF decomposition: CSJDQV and SDP
  - Problem: Checking JP $\rightarrow$ C requires a join!
- Dependency preserving decomposition (Intuitive):
  - If R is decomposed into X, Y and Z, and we enforce the FDs that hold on X, on Y and on Z, then all FDs that were given to hold on R must also hold. *(Avoids Problem (3))*

What FD on a decomposition?

- *Projection of set of FDs F*: If R is decomposed into X, ... the projection of F onto X (denoted F_X) is the set of FDs $U \rightarrow V$ in F^+ (*closure of F*) such that U, V are in X.

Dependency Preserving Decompositions (Contd.)

- Decomposition of R into X and Y is *dependency preserving* if $(F_X \text{ union } F_Y)^+ = F^+$
  - i.e., if we consider only dependencies in the closure F^+ that can be checked in X without considering Y, and in Y without considering X, these imply all dependencies in F^+.
- Important to consider F^+, not F, in this definition:
  - ABC, $A \rightarrow B$, $B \rightarrow C$, $C \rightarrow A$, decomposed into AB and BC.
  - Is this dependency preserving? Is $C \rightarrow A$ preserved?????
- Dependency preserving does not imply lossless join:
  - ABC, $A \rightarrow B$, decomposed into AB and BC.
- And vice-versa!

Another example

- Assume CSJDPQV is decomposed into

  SDP, JS, CJDQV

  It is not dependency preserving

  w.r.t. the FDs: $JP \rightarrow C$, $SD \rightarrow P$ and $J \rightarrow S$.
- However, it is a lossless join decomposition.
- In this case, adding JPC to the collection of relations gives us a dependency preserving decomposition.
- JPC tuples stored only for checking FD!

Summary of BCNF

- If a relation is in BCNF, it is free of redundancies that can be detected using FDs. Thus, trying to ensure that all relations are in BCNF is a good heuristic.
- If a relation is not in BCNF, we can try to decompose it into a collection of BCNF relations.
  - It is always possible to decompose a relation into a set of relations that are in BCNF such that:
    - the decomposition is lossless
    - it may not be possible to preserve dependencies.

Next: Third Normal Form

- There are some situations where
  - BCNF is not dependency preserving, and
  - efficient checking for FD violation on updates is important
- Solution: define a weaker normal form, called Third Normal Form (3NF)
  - Allows some redundancy (with resultant problems; we will see examples later)
  - But functional dependencies can be checked on individual relations without computing a join.
  - There is always a lossless-join, dependency-preserving decomposition into 3NF.

Third Normal Form (3NF)

- If R is in BCNF, obviously in 3NF.
- If R is in 3NF, some redundancy is possible. It is a compromise, used when BCNF not achievable (e.g., no "good" decomposition, or performance considerations).
  - *Lossless-join, dependency-preserving decomposition of R into a collection of 3NF relations always possible.*

3NF

- Relation R with FDs *F* is in 3NF if, for each FD $X \rightarrow A$ ($X \in R$ and $A \in R$) in *F*, one of the following statements is true:
  - $A \in X$ (trivial FD), or
  - X is a superkey, or
  - A is part of some key for R

If one of these two is satisfied for ALL FDs, then R is in BCNF

Not just superkey! (why not?)

What Does 3NF Achieve?

- If 3NF is violated by X→A, one of the following holds:
  - X is a subset of some key K (partial redundancy)
    - We store (X, A) pairs redundantly.
  - X is not a proper subset of any key.
    - There is a chain of FDs K → X → A, which means that we cannot associate an X value with a K value unless we also associate an A value with an X value.
- But: even if reln is in 3NF, these problems could arise.
  - e.g., Reserves SBDC (sid, bid, date, credit_card). Keys are SBD, CBD. FD = {S →C, C →S}. R is in 3NF, but for each reservation of sailor S, same (S, C) pair is stored.
- Thus, 3NF is indeed a compromise relative to BCNF.

Decomposition into 3NF

- Obviously, the algorithm for lossless join decomp into BCNF can be used to obtain a lossless join decomp into 3NF (typically, can stop earlier).
- To ensure dependency preservation, one idea:
  - If $X \rightarrow Y$ is not preserved, add relation XY.
  - Problem is that XY may violate 3NF!
- Refinement: Instead of the given set of FDs F, use a *minimal cover for F*.

Minimal Cover for a Set of FDs

- *Minimal cover* G for a set of FDs F:
  - Closure of F = closure of G.
  - Right hand side of each FD in G is a single attribute.
  - If we modify G by deleting an FD or by deleting attributes from an FD in G, the closure changes.
- Intuitively, every FD in G is needed, and "*as small as possible*" in order to get the same closure as F.

Obtaining Minimal Cover

- Step 1: Put the FDs in a standard form (i.e. right-hand side should contain only single attribute)
- Step 2: Minimize the left side of each FD
- Step 3: Delete redundant FDs

- Find minimal cover for F = {ABH → CK, A → D, C → E, BGH → L, L → AD, E → L, BH → E}

- Step 1: Make RHS of each FD into a single attribute:

$$F = \{ABH \rightarrow C, ABH \rightarrow K, A \rightarrow D, C \rightarrow E, BGH \rightarrow L, L \rightarrow A, L \rightarrow D, E \rightarrow L, BH \rightarrow E\}$$

- F = {ABH → C, ABH → K, A → D, C → E, BGH → L, L → A, L → D, E → L, BH → E}

- Step 2: Eliminate redundant attributes from LHS, e.g. Can an attribute be deleted from ABH → C?
  - Compute (AB)+, (BH)+, (AH)+ and see if any of them contains C. (Why?)
  - (AB)+ = ABD, (BH)+ = ABCDEHKL, (AH)+ = ADH. Since C ∈ (BH)+, BH → C is entailed by F. So A is redundant in ABH → C. Similarly, A is also redundant in ABH → K. Check further to see if B or H is redundant as well.
  - Similarly, for BGH → L, G is redundant since L ∈ (BH)+.
  - F = {BH → C, BH → K, A → D, C → E, BH → L, L → A, L → D, E → L, BH → E}

- F = {BH → C, BH → K, A → D, C → E, BH → L, L → A, L → D, E → L, BH → E}

- Step 3: Delete redundant FDs from F.
  - If F – {*f*} infers *f*, then *f* is redundant, i.e. if *f* is X → A, then check if X+ using F – *f* still contains A. If it does, then it means X → A can be inferred by other FDs.
  - E.g. For BH → L, (BH)+ (not using BH → L) = ACDEKL, which contains L. This means BH → L can be inferred by other FDs, so it's a redundant FD.
  - In fact, BH → L can be inferred by BH → E, E → L.
  - Check other FDs using the same algorithm.

- Note: the order of Step 2 and Step 3 should not be exchanged.

What to do with Minimal Cover?

- After obtaining the minimal cover, for each FD $X \rightarrow A$ in the minimal cover that is not preserved, create a table consisting of XA (so we can check dependency in this new table, i.e. dependency is preserved).
- Why is this new table guaranteed to be in 3NF (whereas if we created the new table from F, it might not?)
  - Since $X \rightarrow A$ is in the minimal cover, $Y \rightarrow A$ does not hold for any Y that is a strict subset of X.
    - So X is a key for XA (satisfies condition #2)
    - If any other dependencies hold over XA, the right side can involve only attributes in X because A is a single attribute (satisfies condition #3).

Comparison of BCNF and 3NF

- It is always possible to decompose a relation into a set of relations that are in 3NF such that:
  - the decomposition is lossless
  - the dependencies are preserved
- It is always possible to decompose a relation into a set of relations that are in BCNF such that:
  - the decomposition is lossless
  - it may not be possible to preserve dependencies.

Normalization Review

- Identify all FD's in F^+
- Identify candidate keys
- Identify (strongest, or specific) normal forms
  - BCNF, 3NF
- Schema decomposition
  - When to decompose
  - How to check if a decomposition is lossless-join and/or dependency preserving
    - Use projection of F^+ to check for dependency preservation
  - Decompose into:
    - Lossless-join
    - Dependency preserving
      - Use minimal cover

Normalization Theory - Practice Questions

Example

A	B	C
1	1	2
1	1	3
2	2	3
2	2	2

FDs with A as the left side:	**Satisfied by the relation?**
$A \to A$	Yes (trivial FD)
$A \to B$	Yes
$A \to C$	No: tuples 1&2
$AB \to A$	Yes (trivial FD)
$AC \to B$	Yes

Example

Let F={ A → BC, B →C }. Is C →AB in F^+?

Answer: No. Either of the following 2 reasons is ok:

Reason 1) C^+=C, and does not include AB.

Reason 2) We can find a relation instance such that it satisfies F but does not satisfy C → AB.

A	B	C
1	1	2
2	1	2

List all the <u>non-trivial</u> FDs in F^+

- Given $F = \{ A \rightarrow B, B \rightarrow C \}$. Compute F^+ (with attributes A, B, C).

	A	B	C	AB	AC	BC	ABC
A		√	√	√	√	√	√
B			√			√	
C							
AB			√		√	√	√
AC		√		√		√	√
BC							
ABC							

Attribute closure
A^+=ABC
B^+=BC
C^+=C
AB^+=ABC
AC^+=ABC
BC^+=BC
ABC^+=ABC

Example

- Given F={ A → B, B → C}. Find a relation that satisfies F:

A	B	C
1	1	2
2	1	2

- Given F={ A → B, B → C}. Find a relation that satisfies F but does not satisfy B → A. Well, the above example suffices.
- Can you find an instance that satisfies F but not A → C? No. Because A → C is in F^+

Examples

R(A, B, C, D, E),
F = {A → B, C → D}

Candidate key: ACE. How do we know?

Intuitively,
-A is not determined by any other attributes (like E), and A has to be in a candidate key (because a candidate key has to determine all the attributes).
- Now if A is in a candidate key, B cannot be in the same candidate key, since we can drop B from the candidate without losing the property of being a "key".
- So B cannot be in a candidate key
- Same reasoning apply to others attributes.

Example

R(A, B, C, D, E),
F = {A → B, C → D} [Same as previous]

Which normal form?

Not in BCNF. This is the case where all attributes in the FDs appear in R. We consider A, and C to see if either is a superkey of not. Obviously, neither A nor C is a superkey, and hence R is not in BCNF. More precisely, we have A → B is in F^+ and non-trivial, but A is not a superkey of R.

Example

R(A, B, C, D, E)
F = {A → B, C → D} [Same as previous]

Which normal form?

We already know that it's not in BCNF.
Not in 3NF either. We have A → B is in F^+ and non-trivial, but A is not a superkey of R. Furthermore, B is not in any candidate key (since the only candidate key is ACE).

Example

- R(A,B,F), F = {AC → E, B → F}.
- Candidate key? AB
- BCNF? No, because of B → F (B is not a superkey).
- 3NF? No, because of B → F (F is not part of a candidate key).

Example

- R(D, C, H, G), F = {A → I, I → A}
- Candidate key? DCHG
- BCNF? Yes
- 3NF? Yes

Example

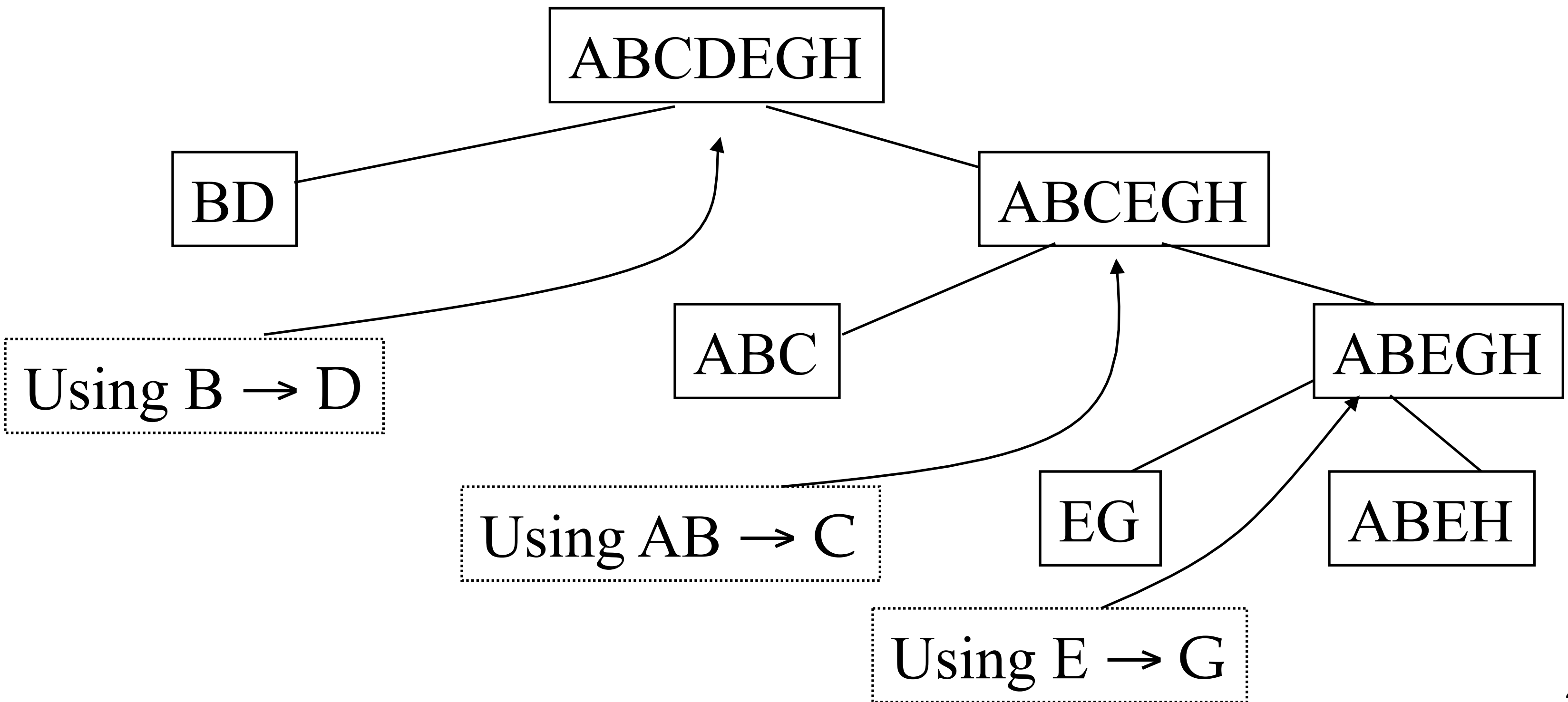
- R(A, B, C, D, E, G, H)
  F={AB → C, AC → B, B → D, BC → A, E → G}

- Candidate keys?
  - H has to be in all candidate keys
  - E has to be in all candidate keys
  - G cannot be in any candidate key (since E is in all candidate keys already).
  - Since AB → C, AC → B and BC → A, we know no candidate key can have ABC together.
  - AEH, BEH, CEH are not superkeys.
  - Try ABEH, ACEH, BCEH. They are all superkeys. And we know they are all candidate keys (since above properties)
  - These are the only candidate keys: (1) each candidate key either contains A, or B, or C since no attributes other than A,B,C determine A, B, C, and (2) if a candidate key contains A, then it must contain either B, or C, and so on.

Example

- Same as previous
- Not in BCNF, not in 3NF
- Decomposition:

R(A, B, C, D, E, G, H)
F={AB → C, AC → B, B → D, BC → A, E → G}



Example

- R(A, B, C, D, E, G, H)
  F={AB → C, AC → B, B → D, BC → A, E → G}
- Decomposition: BD, ABC, EG, ABEH
- Why good decomposition?
  - They are all in BCNF
  - Lossless-join decomposition
    - How do you know this if you don't know how R was decomposed?
  - All dependencies are preserved.

Example

- R(A, B, D, E) decomposed into R1(A, B, D), R2(A, B, E)
- F={AB → DE}
- It is a dependency preserving decomposition!
  - AB → D can be checked in R1
  - AB → E can be checked in R2
  - {AB → DE} is equivalent to {AB → D, AB → E}