CS 484
Data Mining

Classification 3

Some slides are from Professor Eamonn Keogh at UC Riverside

Splitting Based on GINI

- Used in CART, SLIQ, SPRINT.
- When a node p is split into k partitions (children), the quality of split is computed as,

$$GINI_{split} = \sum_{i=1}^{k} \frac{n_i}{n} GINI(i)$$

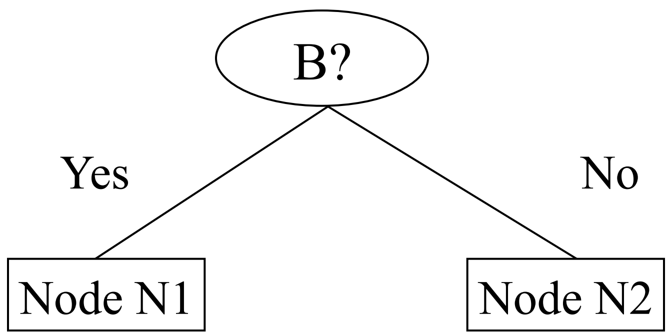
where, n_i = number of records at child i,

n = number of records at node p.

Binary Attributes: Computing GINI Index

- Splits into two partitions
- Effect of Weighing partitions:
  - Larger and purer partitions are sought for.

	Parent
C1	**6**
C2	**6**
Gini = 0.500	



Gini(N1)
$= 1 – (5/7)^2 – (2/7)^2$
= 0.408

Gini(N2)
$= 1 – (1/5)^2 – (4/5)^2$
= 0.32

	N1	N2
C1	**5**	**1**
C2	**2**	**4**
Gini=0.371		

Gini(Children)
= 7/12 * 0.408 +
5/12 * 0.32
= 0.371

Categorical Attributes: Computing Gini Index

- For each distinct value, gather counts for each class in the dataset
- Use the count matrix to make decisions

Multi-way split

	CarType		
	Family	Sports	Luxury
C1	1	2	1
C2	4	1	1
Gini	?		

Two-way split
(find best partition of values)

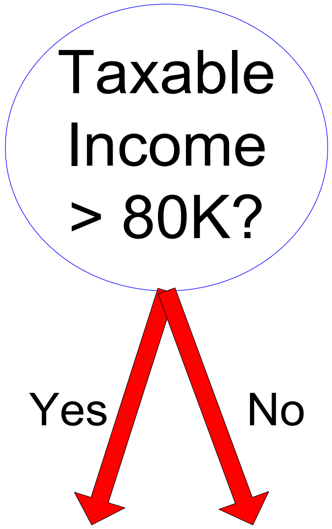
	CarType	
	{Sports, Luxury}	{Family}
C1	3	1
C2	2	4
Gini	?	

	CarType	
	{Sports}	{Family, Luxury}
C1	2	2
C2	1	5
Gini	?	

Continuous Attributes: Computing Gini Index

- Use Binary Decisions based on one value
- Several Choices for the splitting value
  - Number of possible splitting values = Number of distinct values
- Each splitting value has a count matrix associated with it
  - Class counts in each of the partitions, A < v and A ≥ v
- Simple method to choose best v
  - For each v, scan the database to gather count matrix and compute its Gini index
  - Computationally Inefficient! Repetition of work.

Tid	Home Owner	Marital Status	Annual Income	Defaulted
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



Continuous Attributes: Computing Gini Index...

- For efficient computation: for each attribute,
  - Sort the attribute on values
  - Linearly scan these values, each time updating the count matrix and computing gini index
  - Choose the split position that has the least gini index

Defaulted	No	No	No	Yes	Yes	Yes	No	No	No	No
	Income									
Sorted Values →	60	70	75	85	90	95	100	120	125	220

Split Positions →	55		65		72		80		87		92		97		110		122		172		230	
	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>
Yes	0	3	0	3	0	3	0	3	1	2	2	1	3	0	3	0	3	0	3	0	3	0
No	0	7	1	6	2	5	3	4	3	4	3	4	3	4	4	3	5	2	6	1	7	0
Gini	0.420		0.400		0.375		0.343		0.417		0.400		*0.300*		0.343		0.375		0.400		0.420	

Alternative Splitting Criteria based on INFO

- Entropy at a given node t:

$$Entropy(t) = -\sum_j p(j \mid t) \log p(j \mid t)$$

(NOTE: *p(j | t)* is the relative frequency of class j at node t).

  – Measures homogeneity of a node.
    - Maximum ($\log n_c$) when records are equally distributed among all classes implying least information
    - Minimum (0.0) when all records belong to one class, implying most information

  – Entropy based computations are similar to the GINI index computations

Examples for computing Entropy

$$Entropy(t) = -\sum_j p(j\,|\,t)\log_2 p(j\,|\,t)$$

C1	**0**
C2	**6**

P(C1) = 0/6 = 0   P(C2) = 6/6 = 1

Entropy = – 0 log 0 – 1 log 1 = – 0 – 0 = 0

C1	**1**
C2	**5**

P(C1) = 1/6   P(C2) = 5/6

Entropy = – (1/6) $\log_2$ (1/6) – (5/6) $\log_2$ (1/6) = 0.65

C1	**2**
C2	**4**

P(C1) = 2/6   P(C2) = 4/6

Entropy = – (2/6) $\log_2$ (2/6) – (4/6) $\log_2$ (4/6) = 0.92

Splitting Based on INFO...

- Information Gain:

$$GAIN_{split} = Entropy(p) - \left(\sum_{i=1}^{k} \frac{n_i}{n} Entropy(i) \right)$$

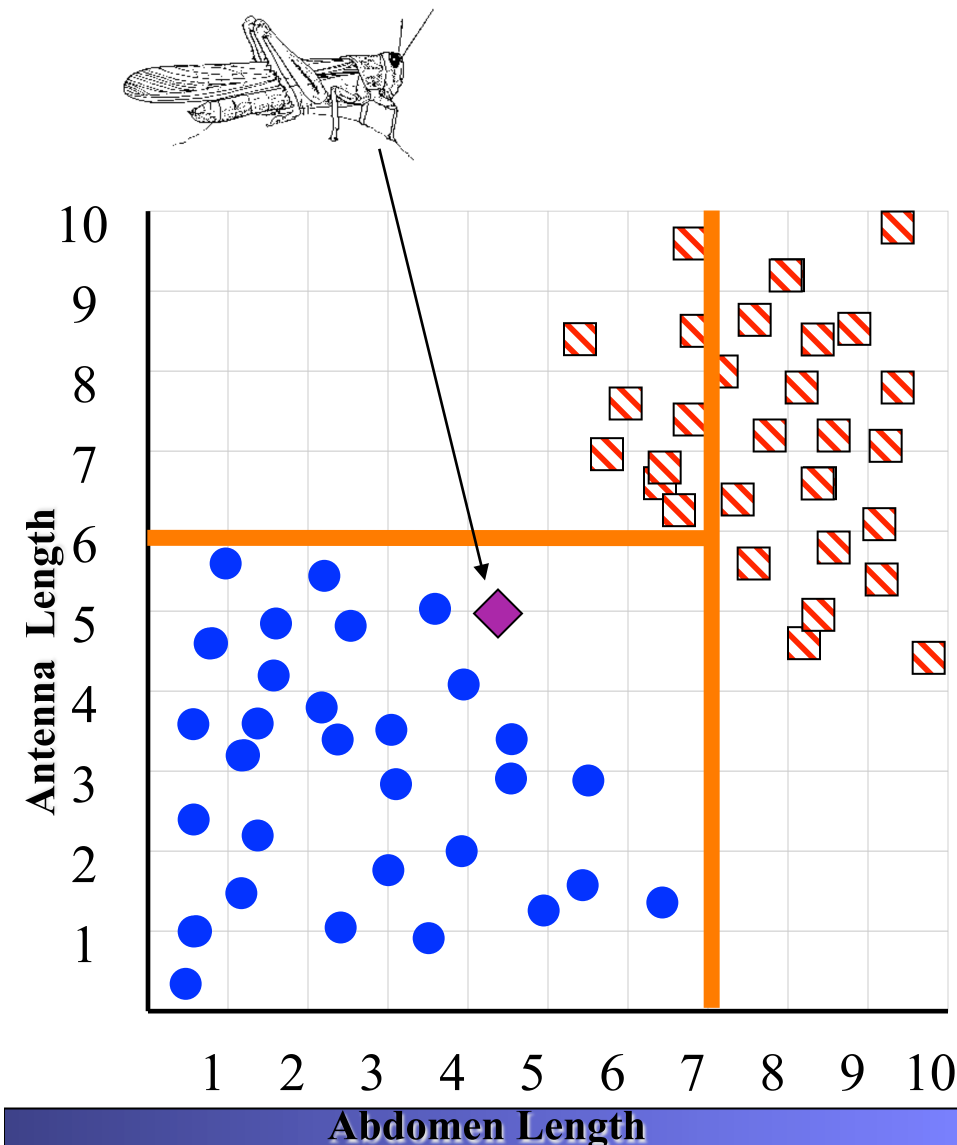
Parent Node, p is split into k partitions;

n_i is number of records in partition i

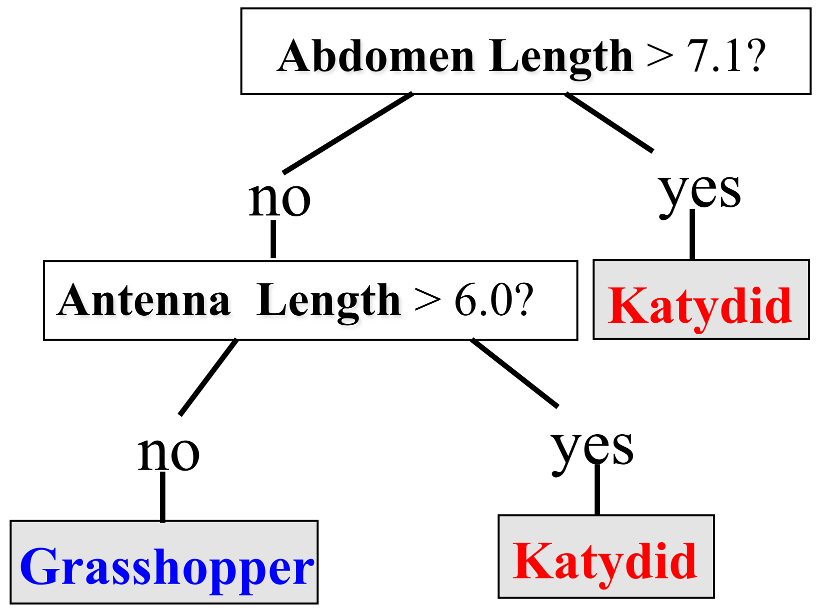
$$\Longrightarrow \quad Gain(split) = E(Parent\ set) - \sum E(all\ child\ sets)$$

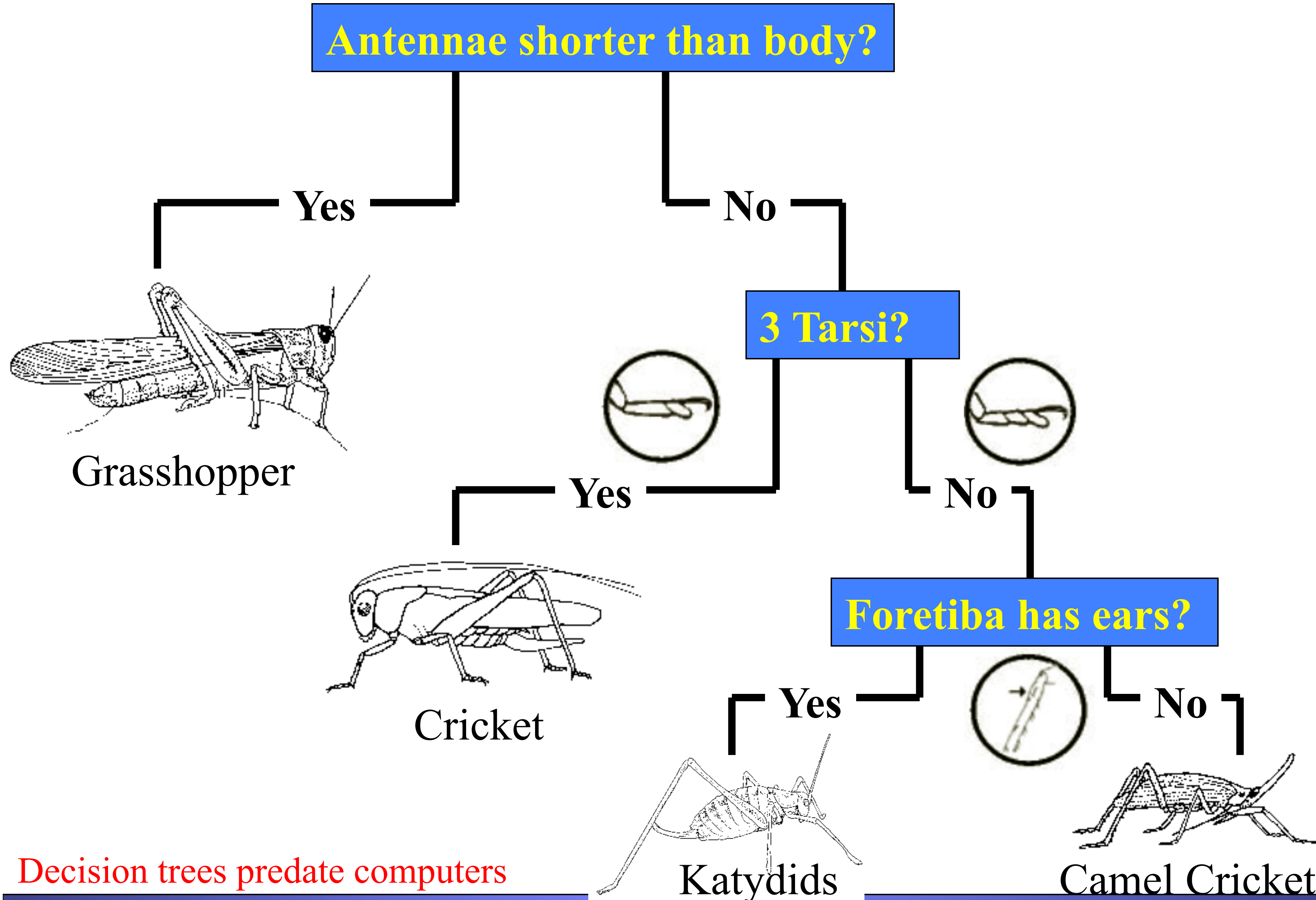
  - Measures Reduction in Entropy achieved because of the split. Choose the split that achieves most reduction (maximizes GAIN)
  - Used in ID3 and C4.5
  - Disadvantage: Tends to prefer splits that result in large number of partitions, each being small but pure.

Back To Our Insect Problem



Ross Quinlan



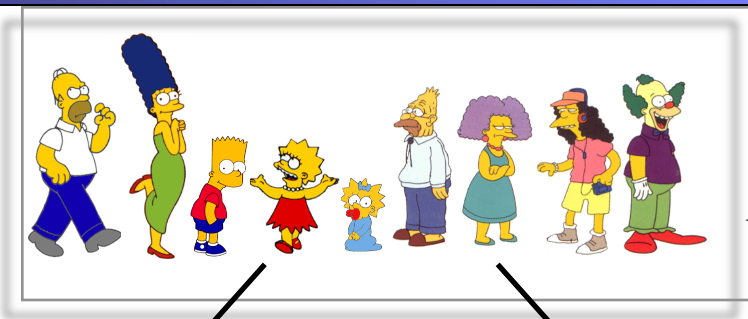


Decision trees predate computers

Person	Hair Length	Weight	Age	Class
Homer	0"	250	36	M
Marge	10"	150	34	F
Bart	2"	90	10	M
Lisa	6"	78	8	F
Maggie	4"	20	1	F
Abe	1"	170	70	M
Selma	8"	160	41	F
Otto	10"	180	38	M
Krusty	6"	200	45	M

Comic	8"	290	38	?

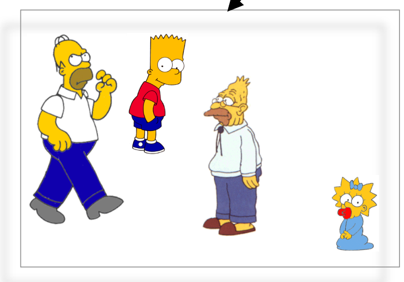
$$Entropy(t) = -\sum_j p(j\,|\,t)\log p(j\,|\,t)$$

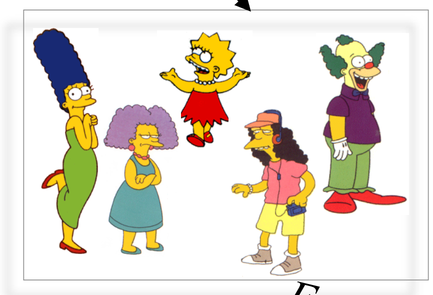


Entropy(4**F**,5**M**) = -(4/9)$\log_2$(4/9) - (5/9)$\log_2$(5/9)
= **0.9911**

yes | no

Hair Length <= 5?





Let us try splitting on *Hair length*

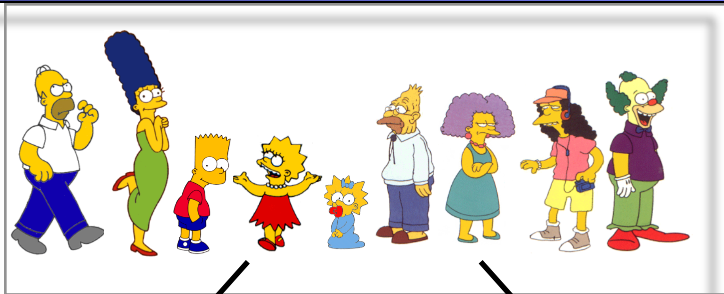
Entropy(1**F**,3**M**) = -(1/4)$\log_2$(1/4) - (3/4)$\log_2$(3/4)
= **0.8113**

Entropy(3**F**,2**M**) = -(3/5)$\log_2$(3/5) - (2/5)$\log_2$(2/5)
= **0.9710**

$$Gain(A) = E(Current\ set) - \sum E(all\ child\ sets)$$

Gain(Hair Length <= 5) = **0.9911** – (4/9 * **0.8113** + 5/9 * **0.9710**) = **0.0911**

$$Entropy(t) = -\sum_j p(j|t) \log p(j|t)$$



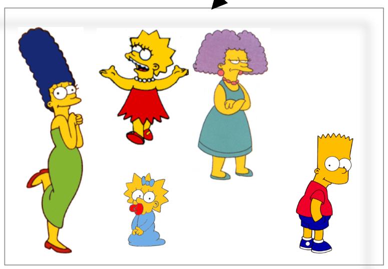
$Entropy(4F,5M) = -(4/9)\log_2(4/9) - (5/9)\log_2(5/9)$

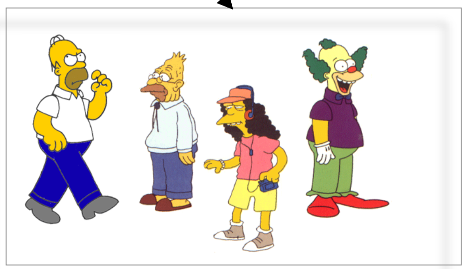
$= 0.9911$

yes

no

Weight <= 160?





Let us try splitting on *Weight*

$Entropy(4F,1M) = -(4/5)\log_2(4/5) - (1/5)\log_2(1/5)$

$= 0.7219$

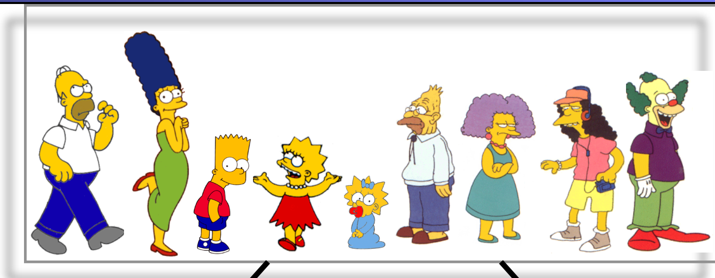
$Entropy(0F,4M) = -(0/4)\log_2(0/4) - (4/4)\log_2(4/4)$

$= 0$

$$Gain(A) = E(Current\ set) - \sum E(all\ child\ sets)$$

$Gain$(Weight <= 160) = **0.9911** – (5/9 * **0.7219** + 4/9 * **0**) = **0.5900**

$$Entropy(t) = -\sum_j p(j|t)\log p(j|t)$$

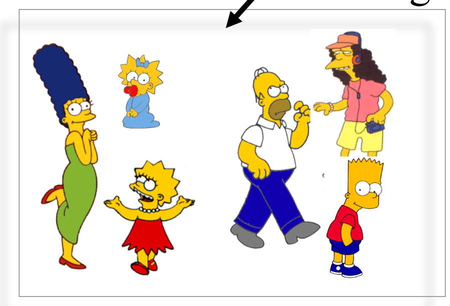


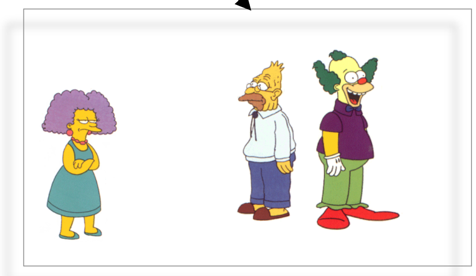
$Entropy(4\mathbf{F},5\mathbf{M}) = -(4/9)\log_2(4/9) - (5/9)\log_2(5/9)$
$= \mathbf{0.9911}$

yes

age <= 40?

no





Let us try splitting on *Age*

$Entropy(3\mathbf{F},3\mathbf{M}) = -(3/6)\log_2(3/6) - (3/6)\log_2(3/6)$
$= \mathbf{1}$

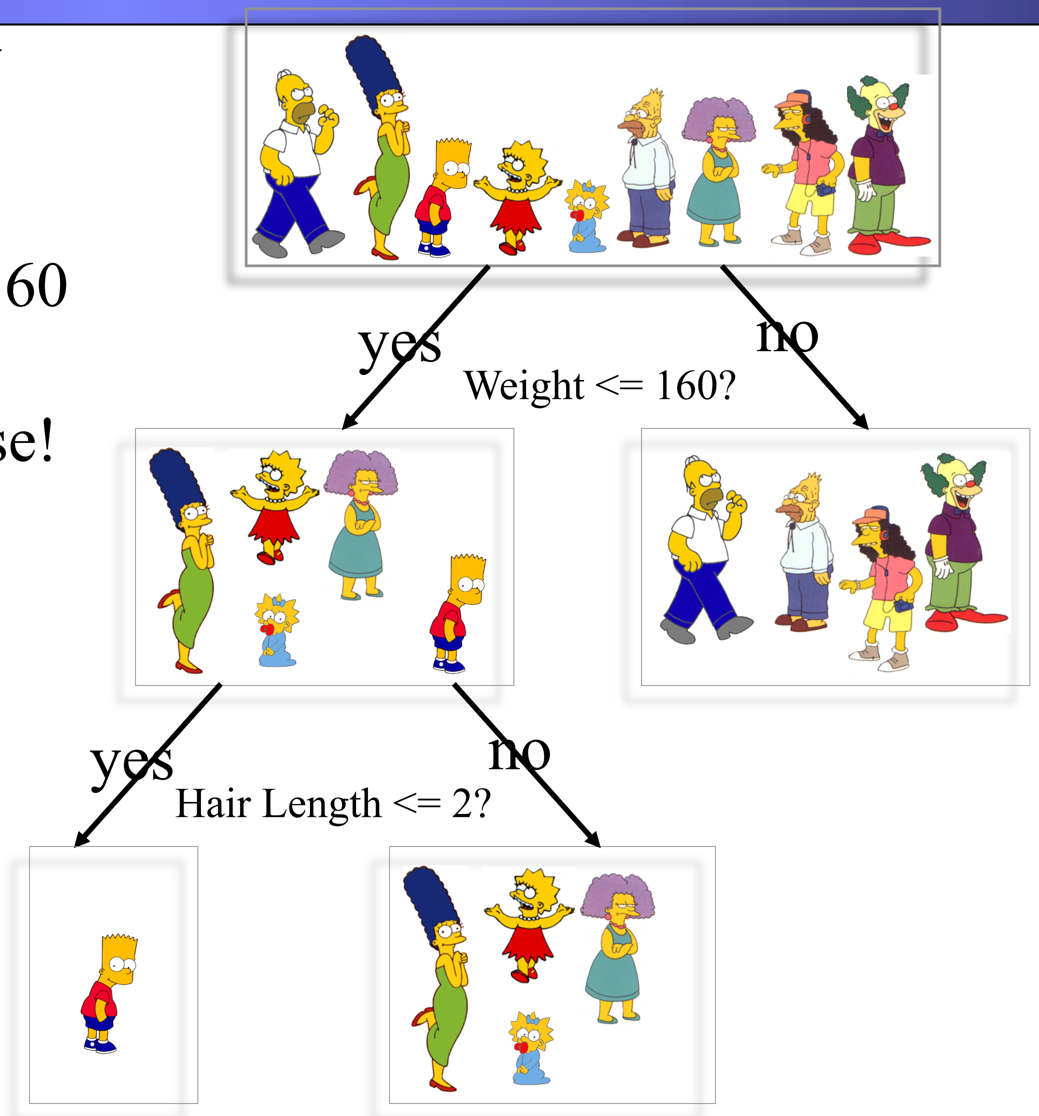
$Entropy(1\mathbf{F},2\mathbf{M}) = -(1/3)\log_2(1/3) - (2/3)\log_2(2/3)$
$= \mathbf{0.9183}$

$$Gain(A) = E(Current\ set) - \sum E(all\ child\ sets)$$

$Gain(\text{Age} <= 40) = \mathbf{0.9911} - (6/9 * \mathbf{1} + 3/9 * \mathbf{0.9183}) = \mathbf{0.0183}$

Of the 3 features we had, *Weight* was best. But while people who weigh over 160 are perfectly classified (as males), the under 160 people are not perfectly classified… So we simply recurse!

This time we find that we can split on *Hair length,* and we are done!



We'll talk more about stopping criteria later.

Splitting Based on INFO...

- Gain Ratio:

$$GainRATIO_{split} = \frac{GAIN_{Split}}{SplitINFO}$$

$$SplitINFO = -\sum_{i=1}^{k} \frac{n_i}{n} \log \frac{n_i}{n}$$

Parent Node, p is split into k partitions

n_i is the number of records in partition i

- Adjusts Information Gain by the entropy of the partitioning (SplitINFO). Higher entropy partitioning (large number of small partitions) is penalized!
- Used in C4.5
- Designed to overcome the disadvantage of Information Gain

Splitting Criteria based on Classification Error

- Classification error at a node t :

$$Error(t) = 1 - \max_j P(j \mid t)$$

- Measures misclassification error made by a node.
  - Maximum ($1 - 1/n_c$) when records are equally distributed among all classes, implying least interesting information
  - Minimum (0.0) when all records belong to one class, implying most interesting information

Examples for Computing Error

$$Error(t) = 1 - \max_i P(i \mid t)$$

C1	**0**
C2	**6**

P(C1) = 0/6 = 0   P(C2) = 6/6 = 1

Error = 1 – max (0, 1) = 1 – 1 = 0

C1	**1**
C2	**5**

P(C1) = 1/6   P(C2) = 5/6

Error = 1 – max (1/6, 5/6) = 1 – 5/6 = 1/6

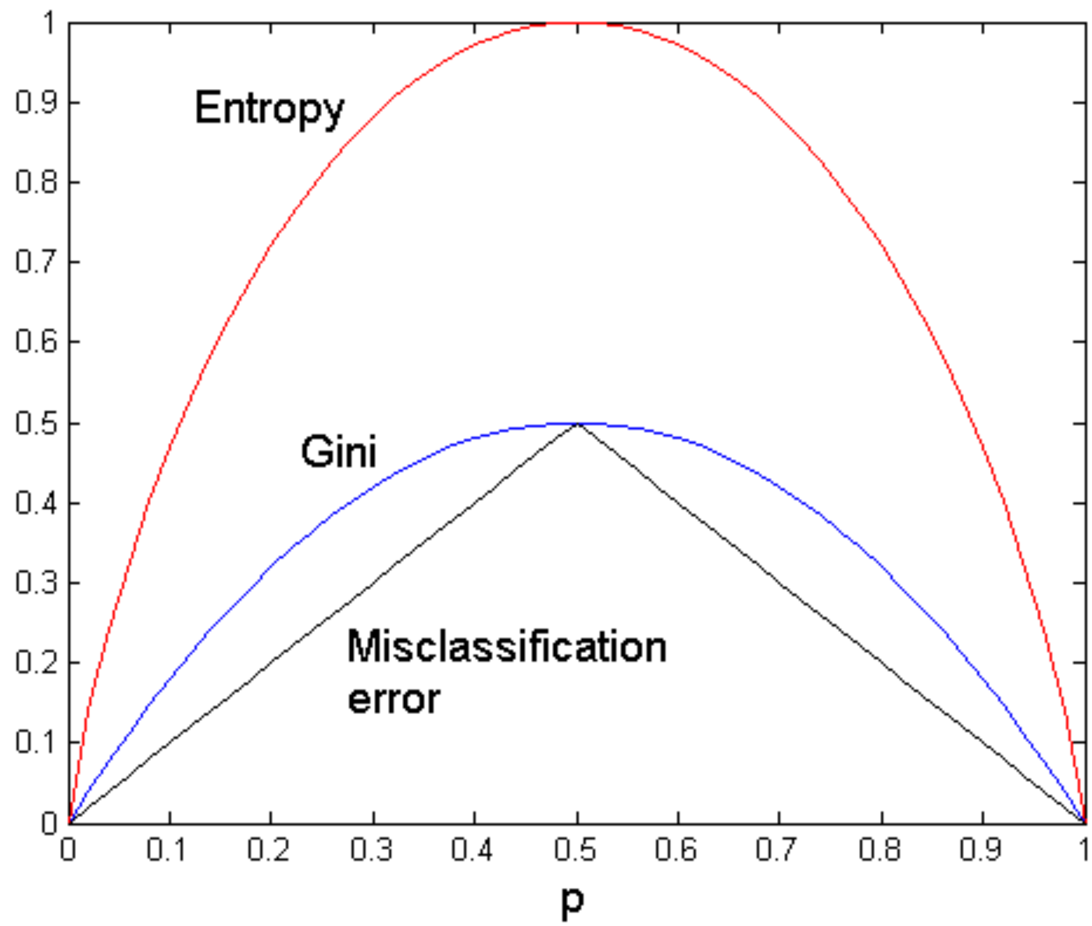
C1	**2**
C2	**4**

P(C1) = 2/6   P(C2) = 4/6

Error = 1 – max (2/6, 4/6) = 1 – 4/6 = 1/3

Comparison among Splitting Criteria

For a 2-class problem:



P refers to the fraction of records that belong to one of the two classes

Tree Induction

- Greedy strategy.
  - Split the records based on an attribute test that optimizes certain criterion.

- Issues
  - Determine how to split the records
    - How to specify the attribute test condition?
    - How to determine the best split?
  - Determine when to stop splitting

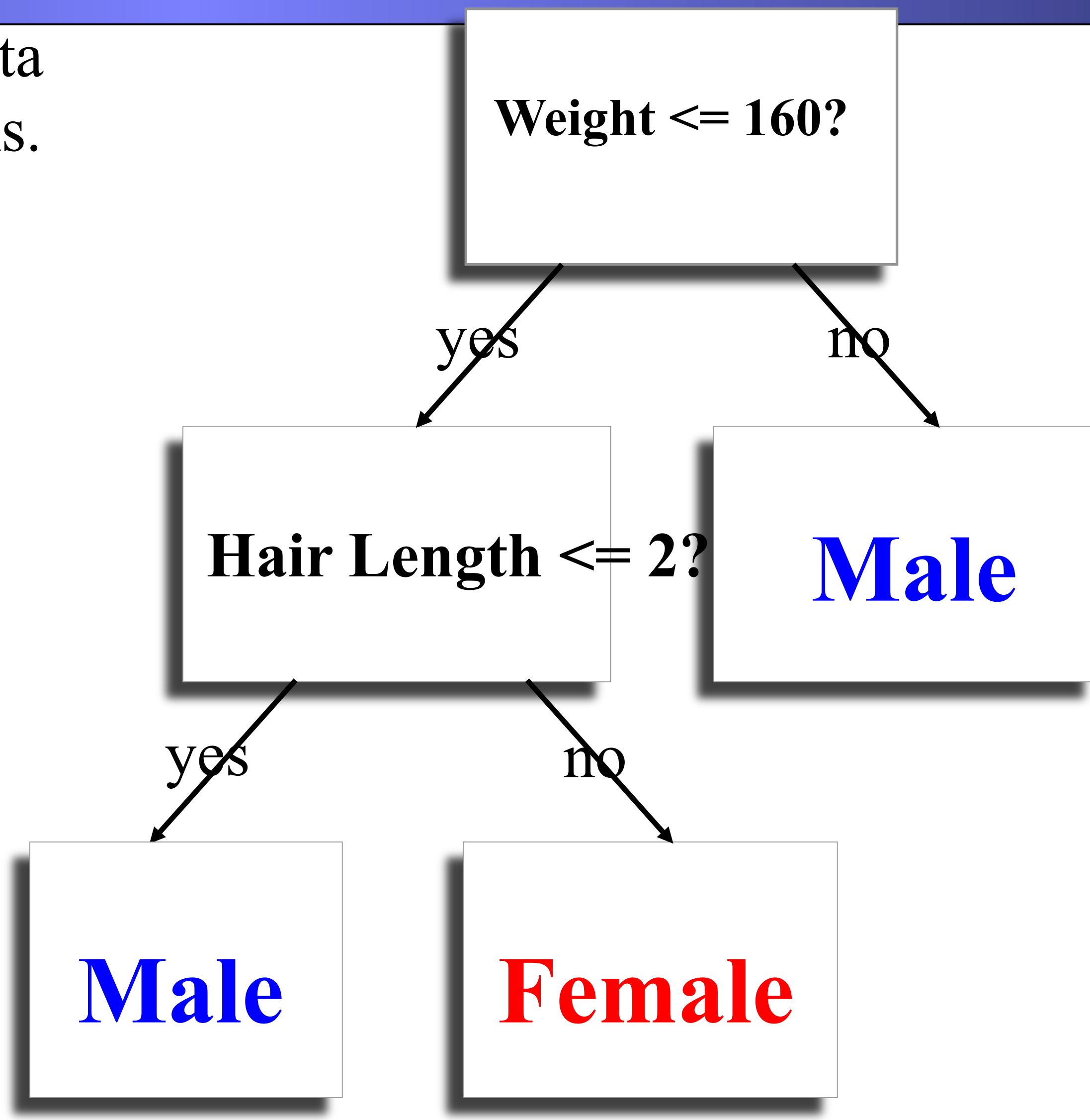
Stopping Criteria for Tree Induction

- Stop expanding a node when all the records belong to the same class
- Stop expanding a node when all the records have similar attribute values
- Early termination (to be discussed later)

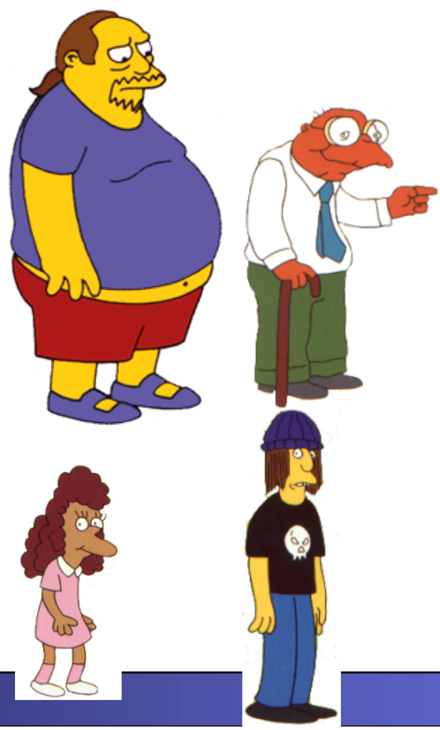
Decision Tree Based Classification

- Advantages:
  - Inexpensive to construct
  - Extremely fast at classifying unknown records
  - Easy to interpret for small-sized trees
  - Accuracy is comparable to other classification techniques for many simple data sets

We don't need to keep the data around, just the test conditions.

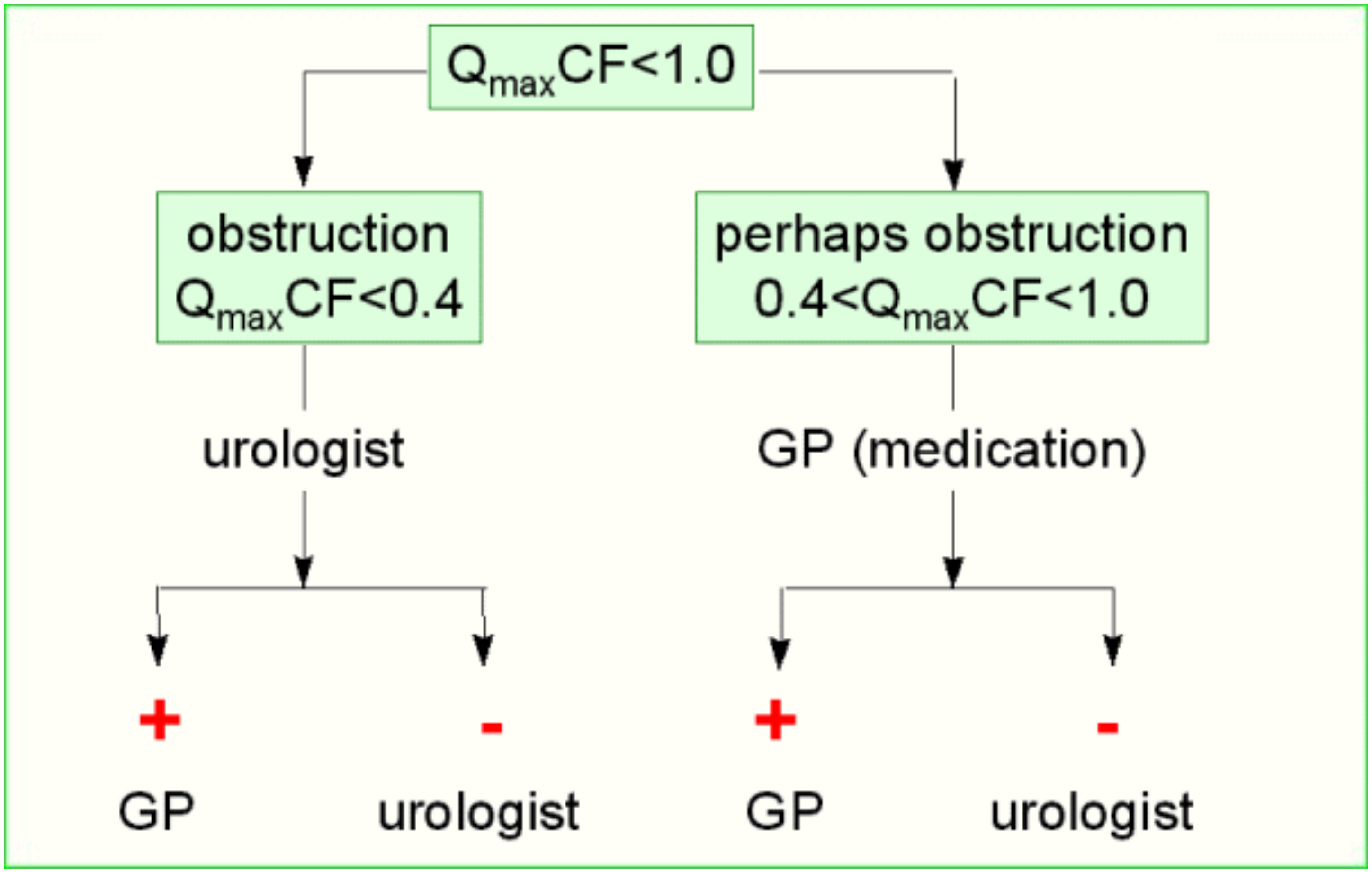


How would these people be classified?



Once we have learned the decision tree, we don't even need a computer!

This decision tree is attached to a medical machine, and is designed to help nurses make decisions about what type of doctor to call.

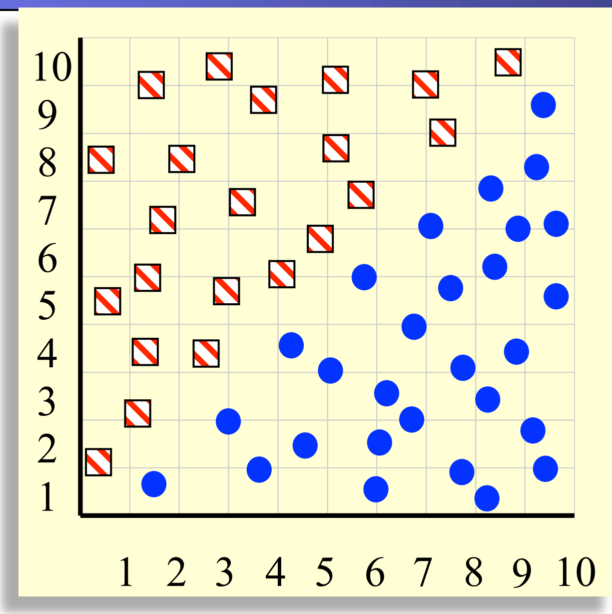


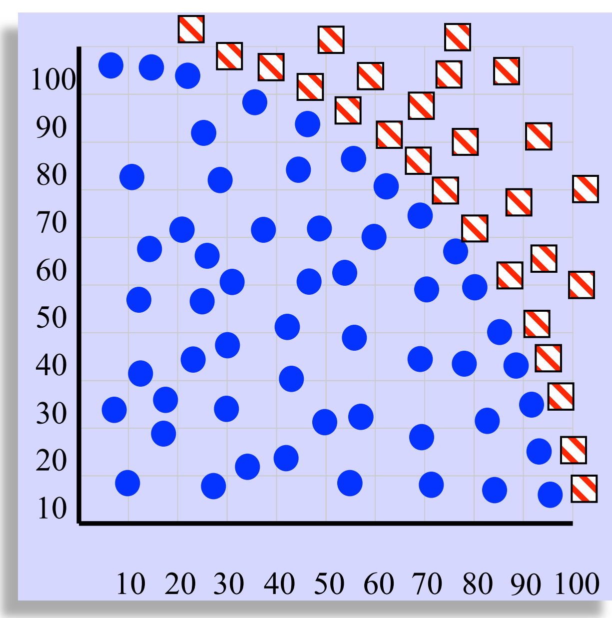
Decision tree for a typical shared-care setting applying the system for the diagnosis of prostatic obstructions.

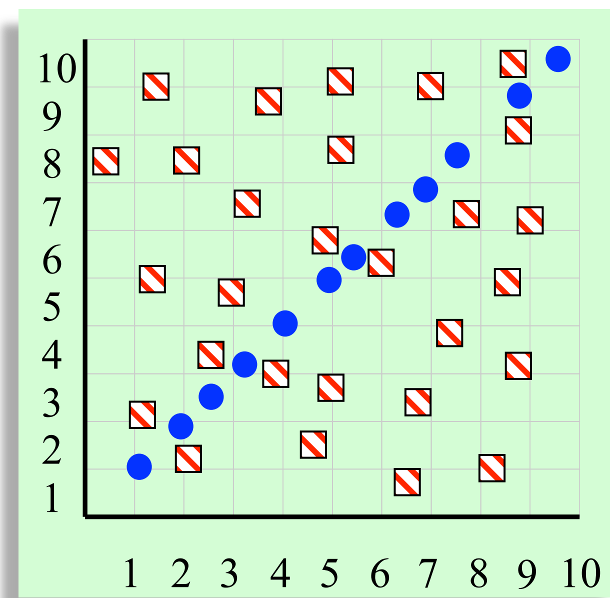
Example: C4.5

- Simple depth-first construction.
- Uses Information Gain
- Sorts Continuous Attributes at each node.
- Needs entire data to fit in memory.
- Unsuitable for Large Datasets.
  - Needs out-of-core sorting.

Which of the "Pigeon Problems" can be solved by a Decision Tree?

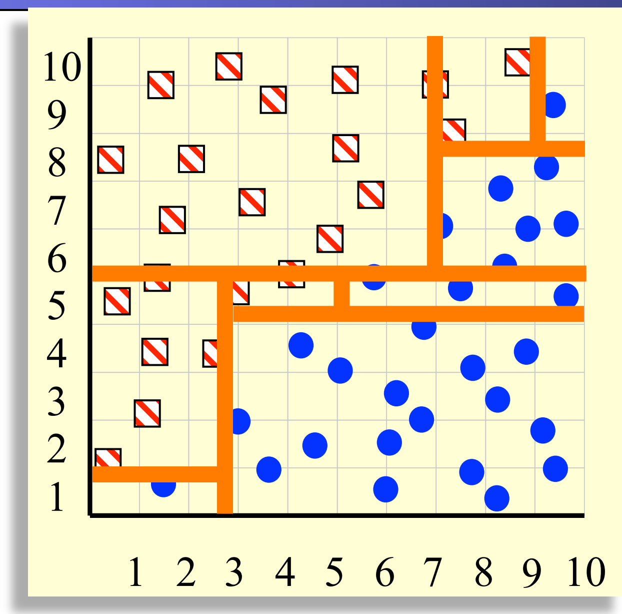




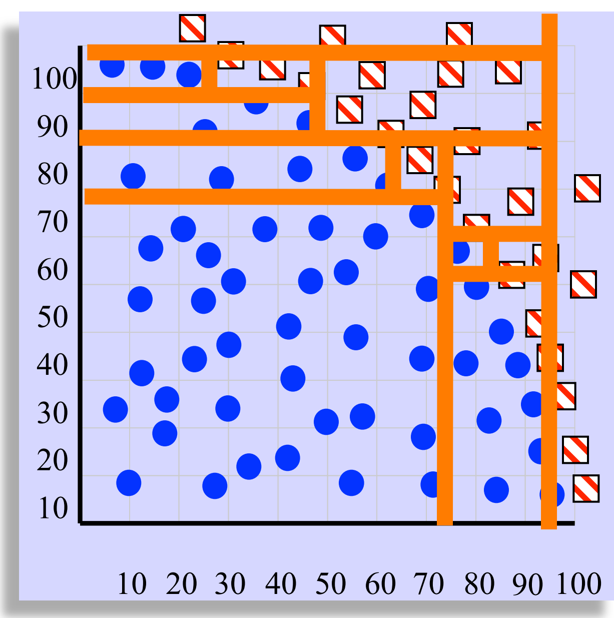


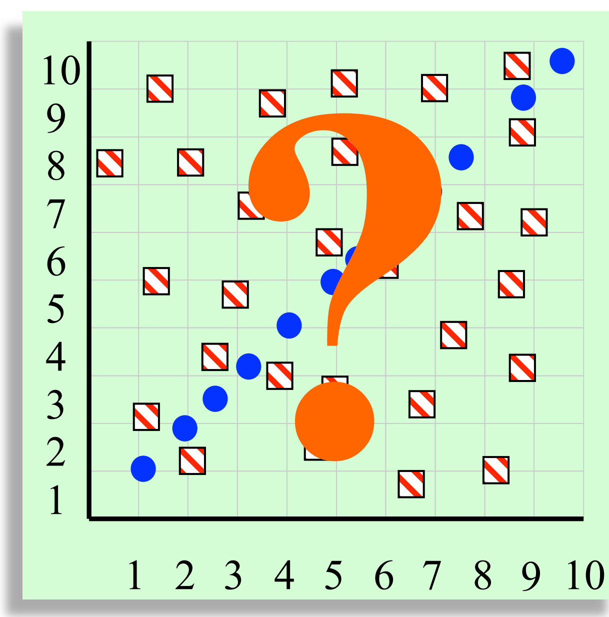
Which of the “Pigeon Problems” can be solved by a Decision Tree?

Deep Bushy Tree
Useless
Deep Bushy Tree



The Decision Tree has a hard time with correlated attributes



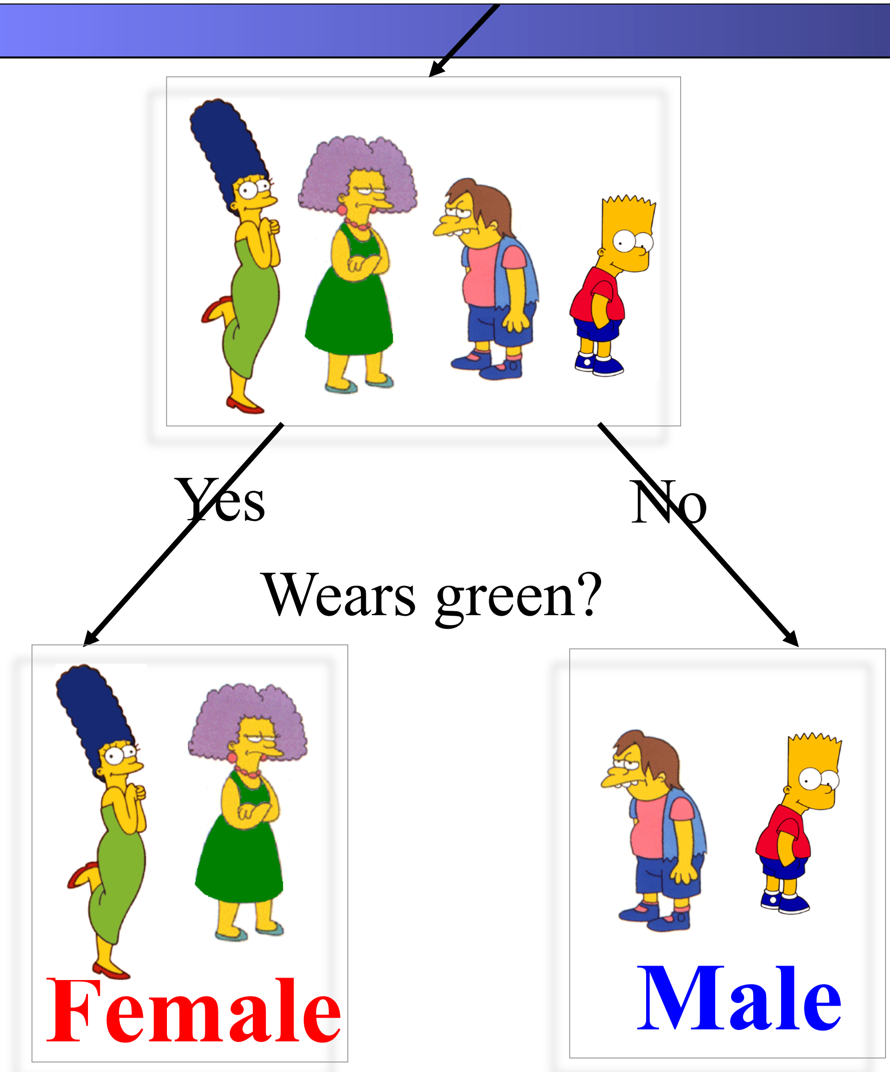


Practical Issues of Classification

- Underfitting and Overfitting
- Missing Values
- Costs of Classification

The previous examples we have seen were performed on small datasets. However with small datasets there is a great danger of overfitting the data…

When you have few data points, there are many possible splitting rules that perfectly classify the data, but will not generalize to future datasets.



For example, the rule "Wears green?" perfectly classifies the data, so does "Mother's name is Jacqueline?", so does "Has blue shoes"…

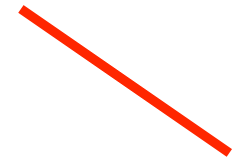
Suppose we need to solve a classification problem

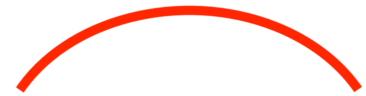
We are not sure if we should use the..

- Simple linear classifier

or the

- Simple quadratic classifier





How do we decide which to use?

We do cross validation (discussed later) and choose the best one.

Predictive Accuracy I

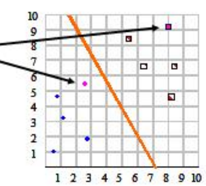
- How do we *estimate* the **accuracy** of our classifier?
  We can use ***K*-fold cross validation**

We divide the dataset into *K* equal sized sections. The algorithm is tested *K* times, each time leaving out one of the *K* section from building the classifier, but using it to *test* the classifier instead

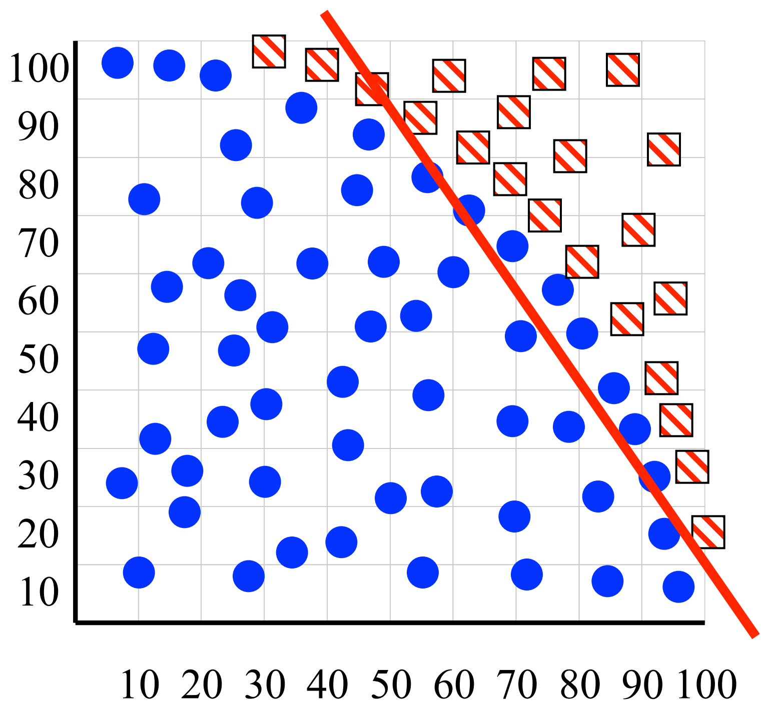
$$\text{Accuracy} = \frac{\text{Number of correct classifications}}{\text{Number of instances in our database}}$$

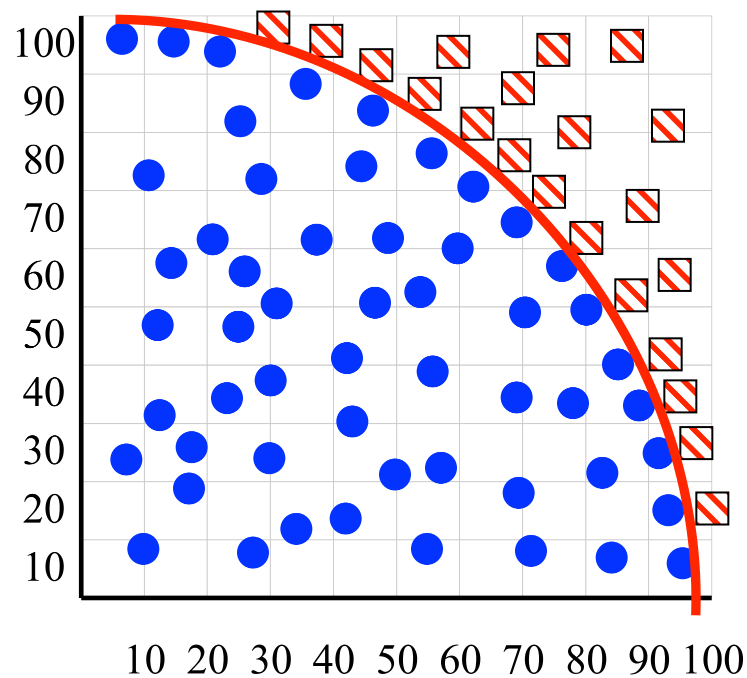
K = 5

Insect ID	Abdomen Length	Antennae Length	Insect Class
1	2.7	5.5	Grasshopper
2	8.0	9.1	Katydid
3	0.9	4.7	Grasshopper
4	1.1	3.1	Grasshopper
5	5.4	8.5	Katydid
6	2.9	1.9	Grasshopper
7	6.1	6.6	Katydid
8	0.5	1.0	Grasshopper
9	8.3	6.6	Katydid
10	8.1	4.7	Katydids

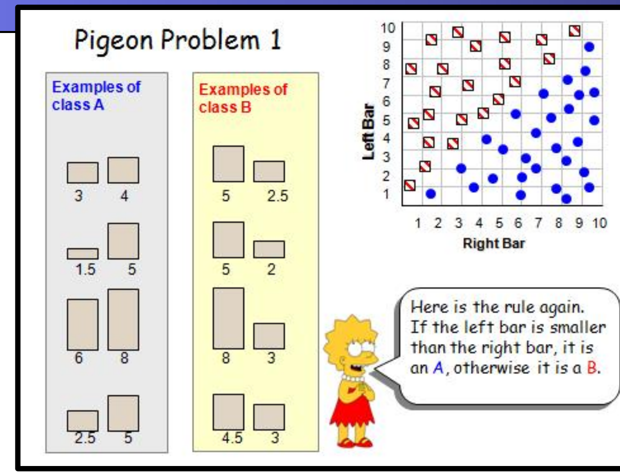


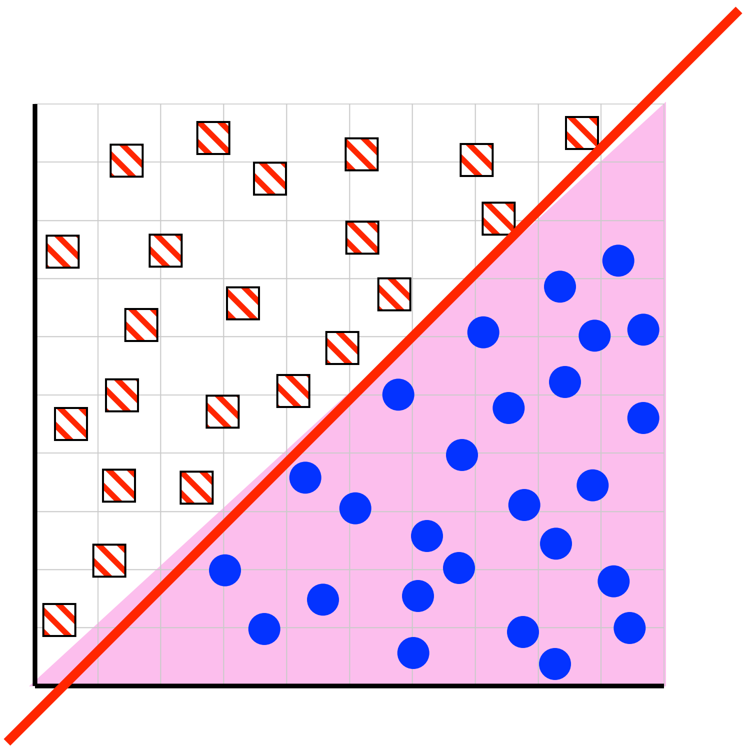
- Simple linear classifier gets 81% accuracy
- Simple quadratic classifier gets 99% accuracy

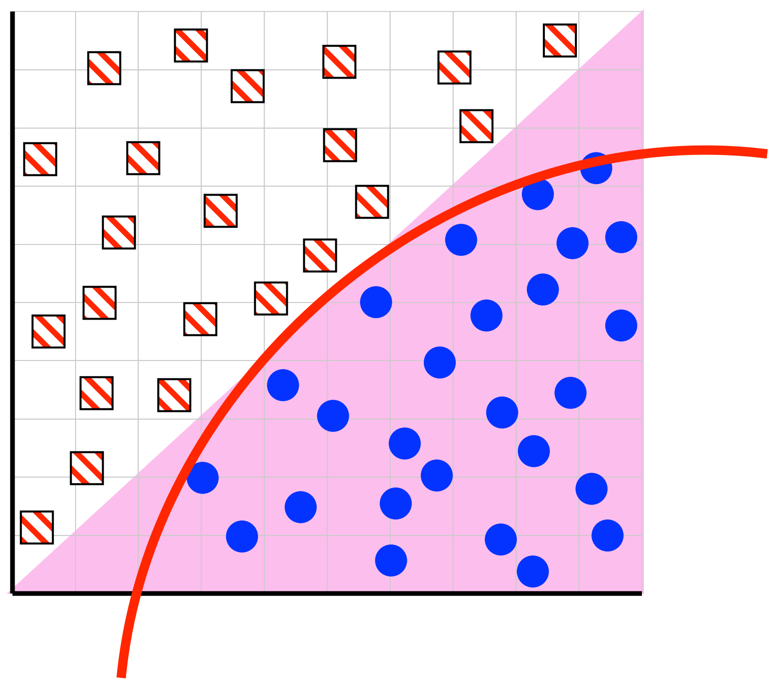




- Simple linear classifier gets 96% accuracy
- Simple quadratic classifier 97% accuracy

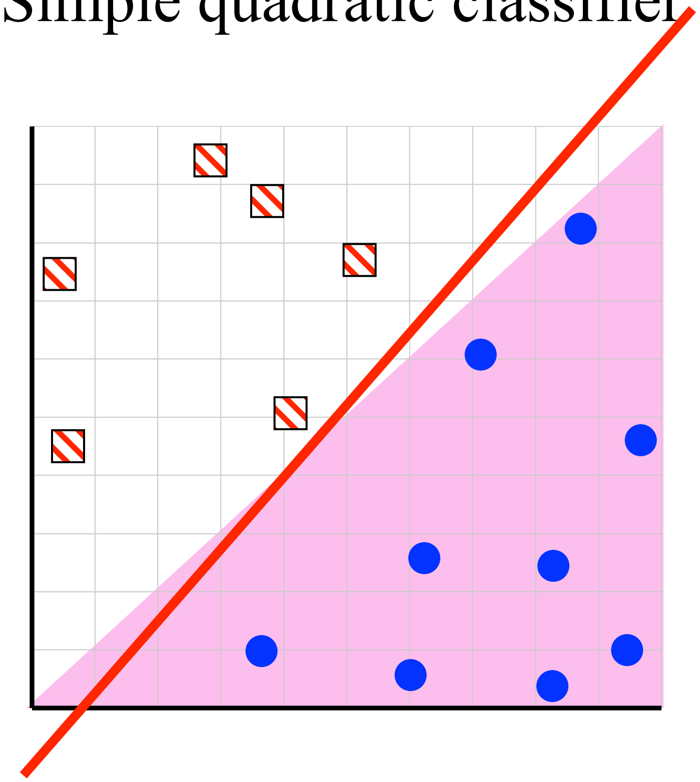


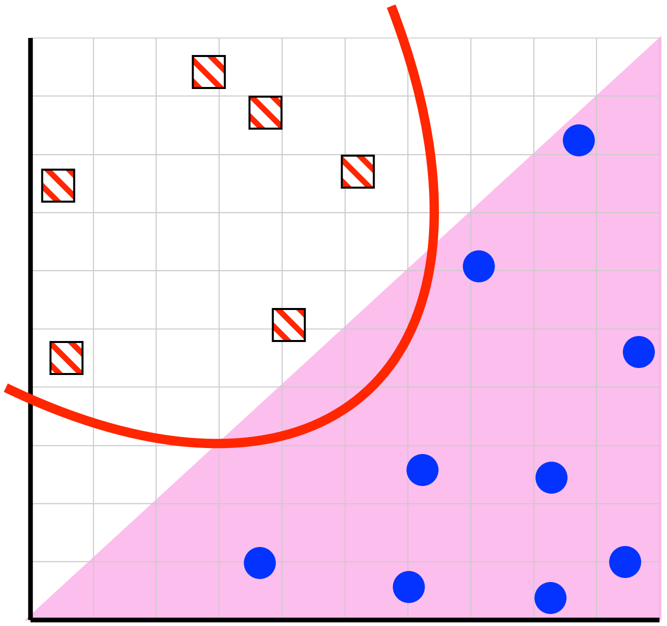




This problem is greatly exacerbated by having too little data

- Simple linear classifier gets 90% accuracy
- Simple quadratic classifier 95% accuracy

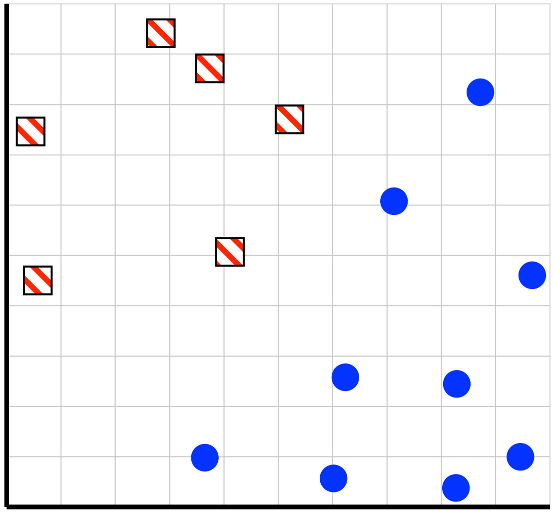




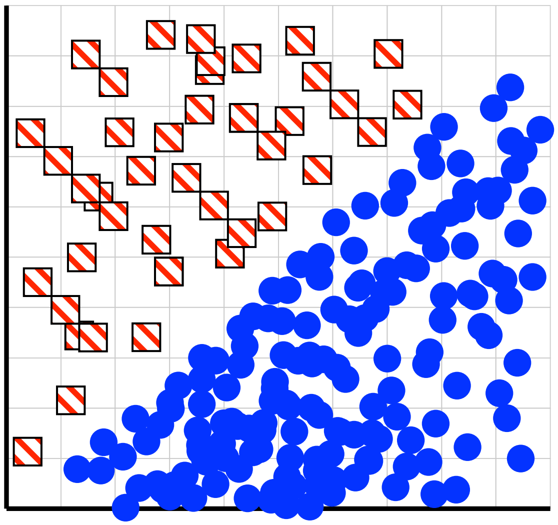
What happens as we have more and more training examples?

The accuracy for all models goes up!
The chance of making a mistake goes down
The cost of the mistake (if made) goes down

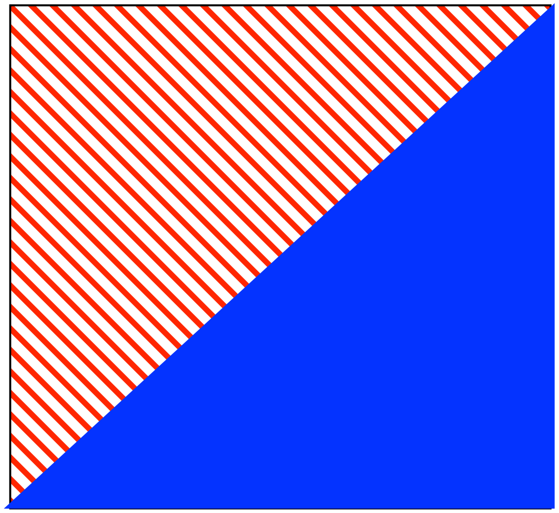
- Simple linear 70% accuracy
- Simple quadratic 90% accuracy



- Simple linear 90% accuracy
- Simple quadratic 95% accuracy



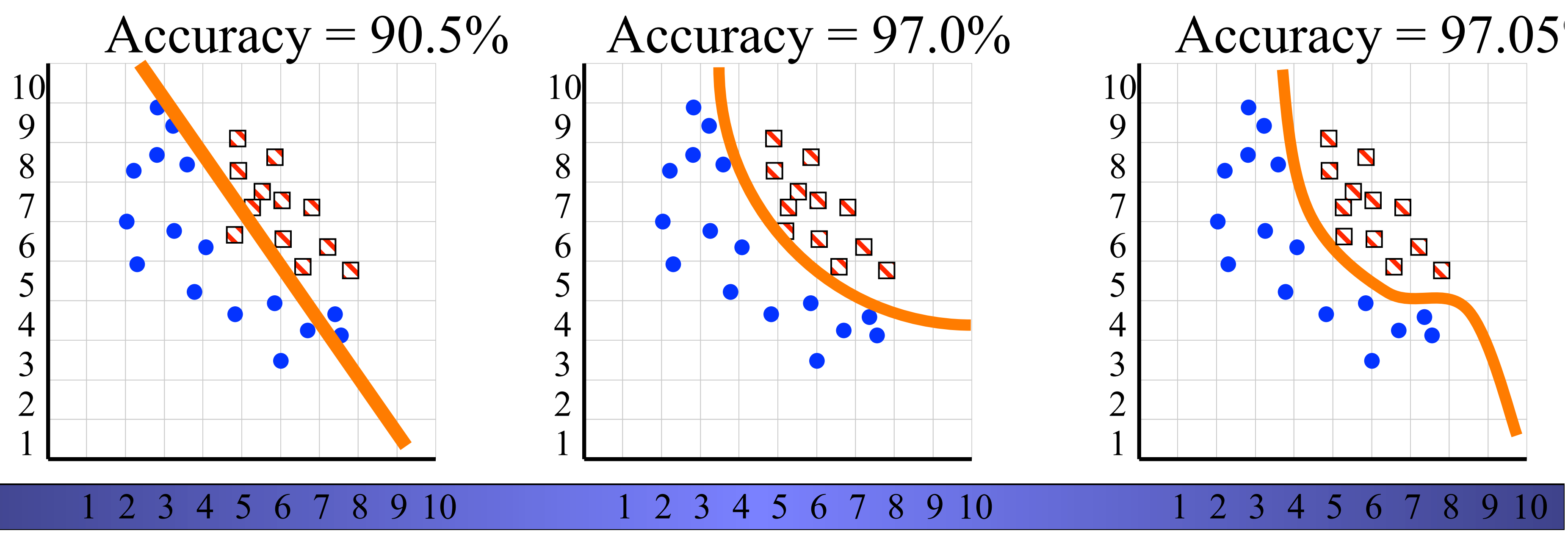
- Simple linear 99% accuracy
- Simple quadratic 99% accuracy



One Solution: Charge Penalty for complex models

- For example, for the simple {polynomial} classifier, we could charge 1% for every increase in the degree of the polynomial

  - Simple linear classifier gets 90.5% accuracy, minus 0, equals 90.5%
  - Simple quadratic classifier 97.0% accuracy, minus 1, equals 96.0%
  - Simple cubic classifier 97.05% accuracy, minus 2, equals 95.05%



One Solution: Charge Penalty for complex models

• For example, for the simple {polynomial} classifier, we could charge 1% for every increase in the degree of the polynomial.

• There are more principled ways to charge penalties
• In particular, there is a technique called **Minimum Description Length** (MDL)