

## Summer Lecture Series 2002

# Introduction to Walsh Analysis

*Alternative Views of the Genetic Algorithm*

Overview and References

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## Overview

In the early 80's, Albert Bethke introduced the use of the *Walsh transform* as a means of understanding what properties of fitness landscapes make it difficult for genetic algorithms to optimize. Bethke was able to demonstrate how this discrete, binary analog of the Fourier transform could be used to gain insight into the kind and quality of information held in particular partitions of the problem space.

A decade later, schema theorists became much more energetically interested in making use of Walsh functions for this and other purposes. The Walsh basis allowed for more rigorous definitions of intuitional concepts (such as “deception”), but also offered the potential for developing formal methods to analyze the properties of problem landscapes in a way that was much more conducive to schema theory: by eliciting *Walsh coefficients*, one can gain exact knowledge of the contributions linear and non-linear components in a schema have on fitness. In this way, direct analysis of these contributing components can be more easily accomplished.

Recent work has shown that the Walsh transform may be helpful for understanding the effects of steps of a GA itself (such as variation) in the context of dynamical system models of GAs. This work suggests that early focus on the application of Walsh analysis to fitness may miss the real power of the Walsh transform to reveal fundamental mathematical properties intrinsic within the algorithm itself.

This lecture attempts to provide a much more general view of what is meant by the term “Walsh Analysis” by suggesting that use of the Walsh transform can be seen as merely a different way to *look* at various aspects of evolutionary algorithms. I define *Walsh analysis* broadly as “Analysis (of a GA) which uses the Walsh transform.” The idea is to suggest that Walsh is not an analysis tool on its own, but is merely a tool which offers a perspective change to be used in conjunction with other analytical methods.

We will begin this lecture by establishing this broader notion of Walsh analysis for contextual purposes. The majority of the lecture will take a close look at the more traditional view of Walsh analysis as a means of analyzing the properties of fitness, then we will spend a little time reviewing the more recent applications of the Walsh transform toward understanding dynamical properties of variation. We will conclude by providing some connecting discussion, as well as some discussion about the utility of the Walsh transform itself.

- Part I: Overview of the Walsh Transform
- Part II: Walsh Analysis of Fitness
- Part III: Walsh Analysis of Mixing Matrices
- Part IV: Conclusions

## References

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